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Heuristic Reasoning about Geospatial Data under Uncertainty

Lorenzo Libertini

Technical University of Munich

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Author: Lorenzo Libertini
Study course: Cartography M.Sc.
Supervisor: Ekaterina Chuprikova, Dr.-Ing. (TU München)
Reviewer: Madalina Gugulica, M.Sc. (TU Dresden)

Chair of the Thesis
Assessment Board: Prof. Dr. Liqiu Meng

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Statement of Authorship

Herewith I declare that I am the sole author of the submitted Master's thesis entitled:

"Heuristic Reasoning about Geospatial Data under Uncertainty"

I have fully referenced the ideas and work of others, whether published or unpublished. Literal or analogous citations are clearly marked as such.

Munich, 08-09-2010 Lorenzo Libertini

Abstract

Research on data uncertainty has witnessed remarkable growth in recent years. Findings across a wide spectrum of knowledge domains demonstrate how humans commonly adopt cognitive biases to navigate through unknown circumstances. In this context, a set of reasoning strategies known as “heuristics” – i.e., logical shortcuts that help individuals to make decisions upon uncertain situations – has been the focus of considerable interest. At the same time, incorporating uncertainty into visualizations has become a crucial issue for GIScience and Cartography. However, there is a notable lack of studies dealing with the *process* of reasoning under uncertainty, particularly in geospatial data. The present thesis aims at filling such a gap by investigating how map-readers make use of heuristics to reason upon geospatial uncertainty, with a specific focus on the visualization of borders, or “borderization”.

In order to accomplish this goal, a set of cartographic techniques to represent the boundaries of two types of natural hazards was tested utilizing a survey with 61 participants. The survey respondents were asked to assess levels of safety and desirability of several housing locations potentially affected by air pollution or avalanches. Maps in the survey varied by boundary type for natural hazard levels (abrupt vs. gradual border), background colour (e.g., red vs. green) and information about areas of uncertain data (extrinsic vs. intrinsic uncertainty). Results were analysed using a mixed quantitative-qualitative approach.

The findings showed the presence of a number of simple heuristics driving users' behaviours. Abrupt borders triggered distance and containment heuristics, whereas gradual boundaries produced more nuanced judgements. Extrinsic uncertainty appeared to increase the overall perception of risk and complicate the use of heuristics by making map choices less straightforward. On the other hand, variations in colour had a more modest impact. Overall, the thesis results can serve to design heuristics-aware visualizations of uncertain boundaries.

Keywords: uncertainty visualization, heuristics, boundaries, cognitive science, user study

Kurzfassung

In den letzten Jahren hat die Forschung über Datenunsicherheit bemerkenswert zugenommen. Erkenntnisse aus verschiedenen Wissensdomänen zeigen, dass Menschen häufig kognitive Verzerrungen nutzen, um sich durch unbekannte Umgebungen zu bewegen. Besonderes wissenschaftliches Interesse haben die sogenannten "Heuristiken" geweckt, das heißt Denkabkürzungen, die für den Einzelnen eine logische Stütze darstellen, um in unsicheren Situationen Entscheidungen treffen zu können. Gleichzeitig wurde die Visualisierung der Unsicherheit zu einer der entscheidendsten Forschungsthemen in der Kartographie und GIScience. Dennoch gibt es erstaunlich wenige Studien über die Denkprozesse der Unsicherheit, vor allem im Bereich der Geodaten. Um diese Lücke zu füllen, wird in dieser Arbeit die Kartennutzung durch Heuristiken um über räumliche Unsicherheit nach-zu-denken erforscht.

Um dieses Ziel zu erreichen, wurden eine Reihe von kartographischen Methoden für die Visualisierung der Grenzen zweier Arten von Naturgefahren getestet. 61 Teilnehmer einer Umfrage wurden gebeten, die Sicherheit, sowie die Attraktivität einiger Wohnstandorte mit potentieller Luftverschmutzungs- oder Lawinen-Gefahr zu beurteilen. Die Karten unterschieden sich durch verschiedene Naturgefahren-Grenzen (abrupt vs. graduell), Hintergrundsfarben (z.B. Rot vs. Grün) und Informationen über Regionen mit unsicheren Daten (extrinsisch vs. intrinsisch). Die Auswertung der Daten erfolgte mittels einem quantitativ-qualitativen Ansatz. Mit den erlangten Ergebnissen konnte man zeigen, dass sich hinter dem Nutzerverhalten einige einfache Heuristiken verbergen. Abrupte Grenzen riefen Entfernungs- und Eindämmungsheuristiken hervor, während graduelle Grenzen nuanciertere Bewertungen bewirkten. Extrinsische Unsicherheit erhöhte die allgemeine Risikowahrnehmung und erschwerte die kartenbasierten Entscheidungen von Nutzern, während die Farbabweichungen nur geringe Auswirkungen zeigten. Insgesamt können die Ergebnisse der Arbeit dazu dienen, heuristikbewusste Visualisierungen unsicherer Grenzen zu entwerfen.

Stichwörter: Visualisierung der Unsicherheit, Heuristiken, Grenzen, Kognitive Wissenschaft, Nutzerstudie

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1. Introduction

1.1 Motivation and problem statement

Dealing with uncertainty in information has been an extensively explored research issue over the last decades. Several unrelated fields, from medicine to archeology or statistics, have dealt with the question of how to incorporate uncertainty in data (Bonneau et al., 2012). The sentence "visualization of uncertainty" alone returns almost a million results on Google Scholar, not including its other possible variations and declinations.

The studies from Tversky & Kahneman have introduced a new line of research on human reasoning and decision-making. The same authors have introduced the concept of prospect theory (see e.g., Tversky & Kahneman, 1979) to explain how cognitive biases drive human choices under uncertain circumstances. These theories have been recognized as landmarks in the history of psychology (see e.g., Morvan & Jenkins, 2017) and this line of cognitive-behavioural studies on decision weighing has acted as a basis for empirical research in fields such as computing (see e.g., Lorkosky & Kreinovich, 2016), risk management in the context of natural hazards (Landry & al., 2019), and even international policy (Herzog, 2019).

Uncertainty is also an especially fundamental issue in the context of cartography, as geospatial data can rarely be assumed to be free from any kind of uncertainty (Yu, 2018) and several types of uncertainty can arise from each stage of the so-called analysis pipeline (Pang et al., 1997). The incorporation of spatial data quality into the final visualization product is crucial in order to deliver the best possible information to the user, as well as supporting the decision-making process (MacEachren et al., 2005). Visualization of uncertainty in a geospatial context has therefore long been a subject of extensive research; both uncertainty itself and corresponding possible visualization techniques

have been typified and taxonomized (MacEachren, 1992; Thomson et al., 2005; Brennen et al., 2017) and have been identified as relevant for visual analytics and human-computer reasoning (MacEachren, 2015; Chuprikova & Meng, 2019).

However, less attention has been dedicated to the concept of *reasoning under uncertainty*, as in the process of reasoning and making assumptions, choices and decisions in a context where data and spatial information are uncertain, or when even the problem itself, as well as its potential outcomes, may be unclear (Chuprikova & Meng, 2019). When making decisions under uncertain conditions, humans tend to base their assumptions on quick and straightforward logical approaches called *heuristics*, which have been categorized (Tversky & Kahneman, 1974) and are supported by underlying cognitive biases that drive the final decisions (Dimara et al., 2014). The lack of research about heuristics related to reasoning under uncertainty, especially within the context of geospatial data, has been identified as a critical gap in research by several authors (e.g., Zuk & Carpendale, 2007; MacEachren, 2015; Kinkeldey et al., 2017).

The present work, sitting at the crossroads between cognitive science and cartography, seeks to contribute to bridge this gap and further explore the issue of heuristics-driven reasoning under uncertainty in a geospatial context, first by designing suitable visualization techniques that take these heuristics into account and then by empirically studying how such techniques impact map readers' decisions.

1.2 Objectives and research questions

The main objective of this thesis is to investigate how different techniques and visual variables can play a role in driving users' decisions under uncertain conditions, taking into account the heuristics that underlie these decisions.

Several sub-objectives make up this main objective:

1. To explore how visual variables influence users' heuristics-driven reasoning under uncertainty in the context of geospatial data, with a specific focus on the issue of geospatial "borderization";
2. To adapt existing visualization techniques to represent "borderization" on selected case studies and investigate users' cognitive biases when reasoning under uncertainty about geospatial data;
3. To build and conduct a user test to interpret how choices of visual variables for uncertainty visualization can affect users' reasoning and map experience.

Each one of these objectives addresses the following research questions:

1. How can the choice of visual variables influence users' heuristics-driven reasoning and decision-making under uncertain conditions in the context of the geospatial "borderization" issue?
2. How can existing visualization techniques be improved and adapted to interpret users' cognitive biases when reasoning under uncertainty in geospatial data?
3. How can a user test, aimed at investigating such techniques in a geospatial context, help evaluate the influence of visual variables on reasoning? How can it be built and administered?

1.3 Research scope

Each sub-objective will require its method.

The first sub-objective will involve an extensive literature review in order to provide a theoretical framework for the following stages.

For the second sub-objective, literature analysis will be the basis to identify suitable case studies to investigate heuristic-driven decision-making processes under uncertain conditions in geospatial contexts. Subsequently, potential visualization techniques aimed at highlighting and controlling heuristic use will be applied to these case studies by analyzing, improving and adapting existing guidelines through the software ArcMap.

The final sub-objective will lead to the design of a user survey aimed at investigating heuristic use in such contexts. This survey will then be administered to a set of relevant respondents after an exploratory pilot test. This survey and its quantitative and qualitative results will then serve to draw conclusions and suggestions on how to evaluate heuristic use under geospatial uncertainty through different visualization techniques, as well as on how to best structure the survey itself.

The issue of "borderization" will be the special focus of the present thesis. Both heuristics and visualization of uncertainty are extremely broad research topics and an exhaustive analysis of these subjects would need to extend far beyond the scope of a single thesis. Even within the specific domain of cartography and geospatial data, these matters can be analyzed through a wide variety of research lenses. Therefore, "borderization" will be the main undercurrent of the present work and will serve to highlight relevant literature, as well as to direct and narrow down the whole study.

1.4 Thesis outline

- Foundations and state-of-the-art: this chapter will outline of the theoretical background and development regarding heuristics and uncertainty visualization, with a special focus on geospatial data. This section will first provide an overview of past and current theories of mental models, cognitive biases and heuristics in human reasoning. Subsequent paragraphs will explain how these theories interlink with

uncertainty visualization and geospatial uncertainty specifically. The final subchapter will include an introduction and motivation for the choice of case studies.

- Methodology: this chapter will present the methodology for the second and third research objectives. The section will explain all the steps implemented to design the maps and the final user test, including the exploratory pilot test.
- Results and discussion: this chapter will list all the findings that resulted from the user test, linking them to each one of the original research objectives. This section will include relevant results on heuristic use in the maps, as well as findings on map design and on the architecture of the user test itself. A final subchapter will provide an essential wrap-up of the most significant findings for each research sub-objective.
- Conclusions and outlook: this chapter will provide a short summary of the previous sections and draw conclusions about the whole thesis, as well as suggesting directions for future work.

2. Foundations and state-of-the-art

2.1 Heuristics and reasoning

In order to navigate through the world, our cognition collects, analyses and filters information coming from the external environment in a continuous stream. The human brain evolved to provide us with a vast number of strategies to make judgements and decisions, as well as reason efficiently and effectively upon any context we might find ourselves called to deal with.

Daniel Kahneman, in his 2011 book "Thinking fast and slow", has summarized previous research findings on the concept of *dual reasoning* (see e.g. Tversky & Kahneman, 1974). According to Kahneman, when faced with problem-solving and decision-making tasks, humans tend to adopt different sets of behaviours that can be classified under two broad categories. The so-called "System 1" is the immediate and quick sequence of suggestions, feelings and thoughtless intuitions that first arise when we face a problem: we can instinctively answer that two plus two equals four without any need for further reasoning. It allows us to filter out unnecessary information and come up with snap judgements that prove effective in most situations. "System 2" introduces doubt, uncertainty and ambiguity; in other words, "System 1" is a synonym for automated thinking, whereas "System 2" is conscious and analytic reasoning that works best for drawing connections and generating calculated conclusions (Ehrlinger et al., 2016). Despite this, "System 2" is time- and energy-consuming and is therefore defined as "lazy" by Kahneman; the unconscious suggestions created under "System 1" are thus turned and further cemented into beliefs in "System 2" and, consequently, used to motivate our deliberate actions (Sanders & Wood, 2019).

Cognitive theory has often favoured a frequentist approach to human reasoning (Griffin et al., 2001), believing that humans behaved as perfectly rational beings who would evaluate uncertain situations through the mathematical laws of probability. This view has been challenged by several opposing currents such as the Bayesian theory, which argues that humans approach probabilities

through subjective judgements; their conclusions do not follow standard rules, but are rather derived from personal experiences and ideas about the likelihood of a certain event (Cosmides & Tooby, 1996).

Drawing upon these premises, cognitive psychology has introduced the so-called model theory (Johnson-Laird, 2010). Rather than postulating that mental processes are akin to mathematical calculus, human brains are assumed to make use of pre-existent mental images, called *models*, to draw and infer conclusions from the available evidence. These models are to be thought of as iconic representations of the problems we are dealing with; therefore, they allow us to retrieve relevant knowledge from our memory about the possible outcomes of such problems and consequently draw timesaving conclusions (Khemlani et al., 2015).

The theories of mental models and subjective probability deeply intertwine with the pre-existing research about inductive and deductive logic. In fact, mental models act as a framework and a starting point for both types of reasoning. Deductive reasoning uses mental models to lead to certain syllogistic conclusions from premises that it assumes as fully valid; inductive reasoning employs mental models to increase semantic information by inferring new possible outcomes from a partially known set of premises (Bara & Bucciarelli, 2000). Therefore, deductive reasoning draws particular conclusions from general statements that necessarily imply them from the start, whereas inductive reasoning leads to the recognition of new connections, rules and properties (Dantlgraber et al., 2019).

Under this view, inductive logic can act as a crucial support when reasoning under uncertain conditions with partially or fully unknown evidence and consequences (Mastropasqua et al., 2010). In fact, humans use mental models to envisage salient possibilities and consequently elicit judgements within probabilistic systems (Johnson-Laird, 2010).

Mental models, however, while assumed to represent reality by those holding them, do not necessarily yield true and rational conclusions (Johnson-Laird, 2010). In this context a set of strategies called *heuristics*, which have received

increasing attention across several knowledge domains over the last decades, plays an essential role.

The idea that humans might not necessarily behave rationally when confronted with uncertainty has been circulating in philosophy since as early as the nineteenth century (Miller & Gelman, 2018). However, it is only in the last decades that biases in reasoning have solidly become a subject of research.

The landmark work of Amos Tversky and Daniel Kahneman (1974) has introduced and widely popularized the concept of heuristics, defined as mental principles that "reduce the complex tasks of assessing probabilities and predicting values to simpler judgemental operations" (Tversky & Kahneman, 1974, p. 1124). In other words, heuristics are mental shortcuts that allow us to analyse a limited amount of relevant information when making decisions and judgements under instances of uncertainty. Therefore, they help us to save time and mental effort but this may happen, in turn, at the expense of accuracy by producing systematic errors (Dale, 2015). The paper from Tversky & Kahneman (1974) presented three broad categories of heuristics:

- *Representativeness*: the intuitive belief that an event might be more likely to happen if it is akin to the mental stereotype that people have of that event. A typical example of the representativeness heuristic is the so-called gambler's fallacy, where e.g., people expect a tail to be highly likely in a coin flip after a series of heads because it would be more representative of the expected probability distribution, despite the two events being independent of each other (Benjamin, 2018);
- *Availability*: the propensity to gauge the likelihood of a phenomenon based on how easy it is to retrieve mental images of instances of that phenomenon. For example, we might feel unsafe on a plane due to a recent much-publicized plane crash (Dimara et al., 2016);
- *Adjustment to an anchor*: the tendency to make final estimations by over-relying on the starting point, as in, the first piece of information that has been received. For example, car salesmen are more likely to negotiate a higher final price for a sale if their first offer is a high starting point and

then they move down, as the consumer might perceive the final price to be more valid (Dale, 2015).

While the study from Tversky & Kahneman (1974) belonged to the field of behavioural economics, heuristics and related cognitive biases have since entered the research discourse in several other knowledge domains. Heuristics have since then been described as rough intuitive equivalents to statistical probabilities that are crucial for everyday reasoning (Nisbett et al., 1983). For example, computer scientists have used the concept of heuristics to explain how human reasoning is fundamentally driven by "fuzzy" and imprecise assumptions that might be difficult to compute (Lorkowski & Kreinovich, 2018). In contrast, medicine has recognized that doctors commonly employ heuristics to take judgement on their patients' health, especially under heavy time constraints (Itri & Patel, 2018).

Tversky and Kahneman have themselves kept expanding their original scope of research. In a later study they introduced the so-called prospect theory, believing that the way individuals are theoretically expected to behave under uncertain conditions and reason upon probabilistic data is different from how they actually behave (Tversky & Kahneman, 1979). They argued that when making choices and assumptions people tend to apply biases that sometimes contrast with those that the literature assumed as standard at the time. Within the paper, they defined *loss aversion* as the tendency to focus on minimizing losses rather than avoiding risk per se. They also described *overweighing disjunction* as a bias that implies paying more attention to the differences between two alternatives over the elements they share. Furthermore, they introduced the concept of *prospect weighing* – the idea that each "prospect" (as in, each expected outcome) is attached to a weighed perceived probability, which may or may not be the same as the officially stated one.

Later on, several other reviews (e.g. Arnott, 2006; Battersby, 2016; Padilla et al., 2018) further broadened the issue of heuristics by examining several other bi-

ases that had not been researched by Tversky and Kahneman, as well as extensively dealing with the underlying cognitive biases that support them. Some of those that have been recurrent in the literature are listed here as examples:

- Base-rate fallacy: a mental bias that leads to judging the probability of an event by disregarding general pieces of information in favour of more event-specific ones; this leads to overestimating the likelihood of the event by ignoring its base rate in the general population (Dale, 2015);
- Affect heuristic: the overreliance on feelings over objective probability judgements (Raue & Scholl, 2018);
- Habit and familiarity heuristic: the tendency to solve known problems with already known and pre-experimented solutions (Arnott, 2006; Dale, 2015).

Many of these biases have become ever-growing subjects of research in information visualization and especially in geospatial visualization. Indeed, research on heuristics and cognitive biases has been recognized in both fields as crucial to developing further understanding of how humans perceive information visually and, therefore, how and why we should choose certain visualizations (e.g. Zuk & Carpendale, 2007; MacEachren, 2015). In the context of visualization perception there is also substantial literature available on the concept of *visual semiotics*, which is the study of how visual variables tend to convey information and, subsequently, how the final users interpret them (MacEachren et al., 2012).

2.2 Visualization of uncertainty in geospatial data: an overview

We can rarely assume data to be completely free from any kind of uncertainty (Yu, 2018) and the inclusion of uncertainty into the visualization has long been the subject of an ever-growing field of research (Chuprikova et al., 2018). Therefore, the incorporation of uncertainty is, as stated, a fundamental issue when it comes to the visualization of any kind of data, be it geospatial or non-geospatial.

Substantial attention has been dedicated to both *visualization of uncertainty*, which means visually representing uncertainty that is already present in the original data, and *uncertainty of visualization*, which is a common by-product of

errors and inaccuracies that might happen all along the process of data collection, transformation and analysis (Gotz et al., 2019). Factors such as accuracy and reliability are defining elements of data quality (Beard & Buttenfield, 1999) and their inclusion into the final visualization product is crucial to support users in a decision-making context (MacEachren, 2005). Uncertainty in data has been taxonomized in its key major categories (Thomson et al., 2005) as well as its visualization and evaluation methods (Zuk & Carpendale, 2007; Bonneau et al., 2014).

Uncertainty is an inherent component of data (Padilla et al., 2020) and the stages of data acquisition, transformation and visualization itself can also generate uncertainty, which Pang et al. (1997; see Fig. 2.1) divide into three main types:

- *statistical* (expressed as a confidence interval of a certain value);
- *error* (the difference between a known value and an estimate);
- *range* (an interval that necessarily includes the value, but without any information on the value itself).

Adding uncertainty information is, therefore, crucial to delivering a clearer picture of the data represented.

The issue of how to visualize uncertainty in geospatial data has been the subject of a vast body of research over the last decades (MacEachren et al., 2005) and the diverse wealth of approaches available has been analysed in several extensive reviews (e.g. Bostrom et al., 2008; Zuk, 2008; Brodlie et al., 2012).

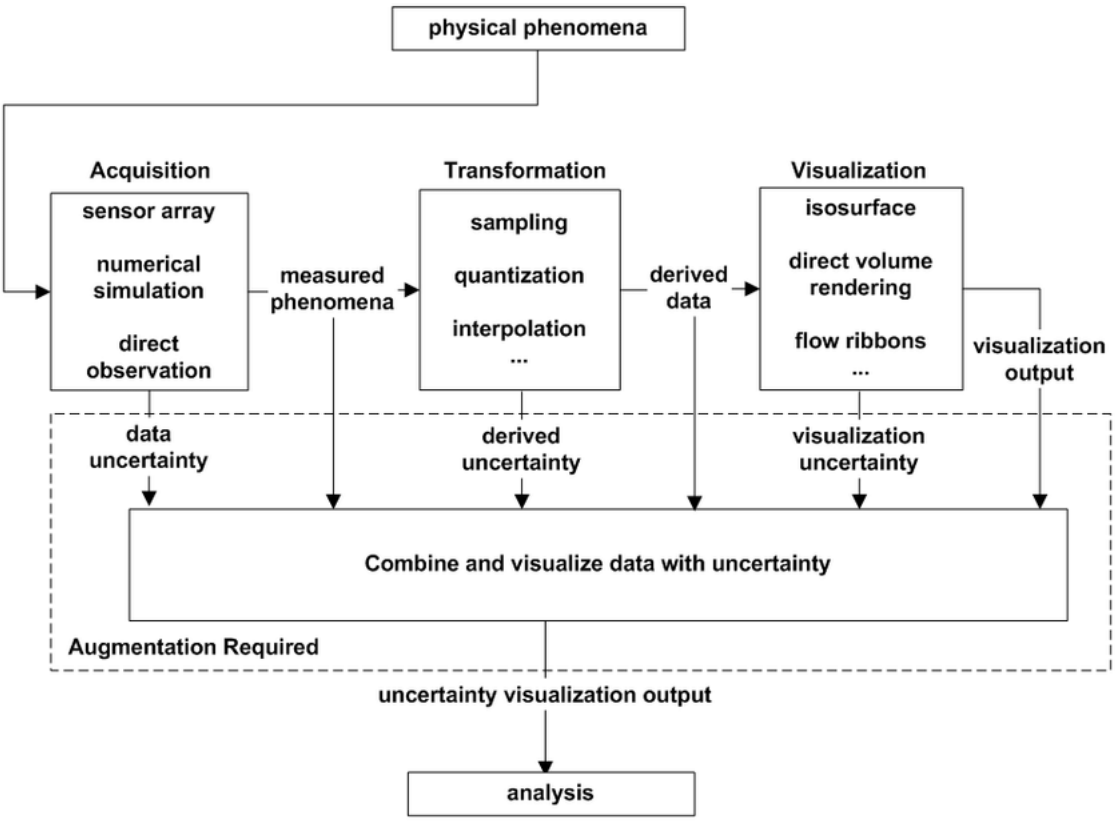


Figure 2.1: Visualization pipeline (Pang et al., 1997). The pipeline describes how all the stages of data collection, analysis and visualization can produce uncertainty.

The work of Bertin (1983) determined seven visual variables (see Fig. 2.2) that make up the main components of any map symbolization. Later on, MacEachren (1992) analysed these findings from the perspective of uncertainty visualization and introduced two new variables that had been ignored by Bertin, namely *colour saturation* (where an unsaturated hue would signify higher uncertainty) and *symbol focus*. MacEachren (1992) further divided the symbol focus into four main typologies: *contour crispness* (or fuzziness of edges), *fill clarity, fog* (in other words, the possibility to create a "foggy" layer between the observer and the map) and *resolution*. He then also explained how each one of these variables could be best used to visualize uncertainty, e.g. by increasing contour fuzziness to display higher uncertainty.

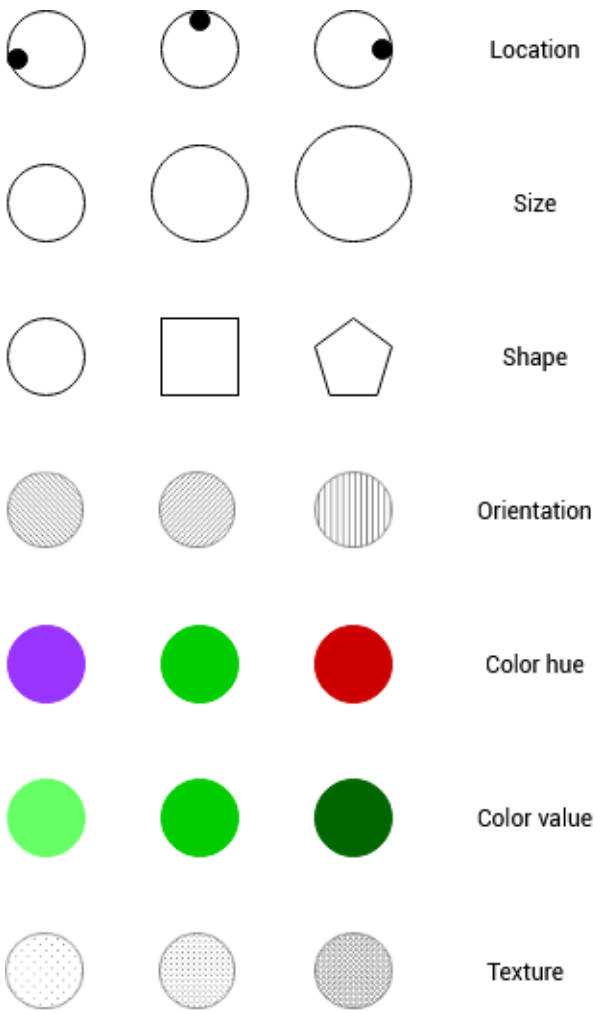


Figure 2.2: Bertin's visual variables. Image redrawn from an adaptation included in Roth, 2017.

MacEachren et al. (2005) expanded these studies by extending the research and taxonomy from Thomson et al. (2005; see Table 2.1), thus categorizing data quality, and subsequent uncertainty, into the five components of lineage (a description of the source material), positional accuracy, attribute accuracy, logical consistency and completeness.

Accuracy/error	Difference between observation and reality
Precision	Exactness of measurement
Completeness	Extent to which info is comprehensive
Consistency	Extent to which info components agree
Lineage	Conduit through which info is passed
Currency/timing	Temporal gaps between occurrence, info collection and use

Credibility	Reliability of info source
Subjectivity	Amount of interpretation or judgement included
Interrelatedness	Source independence from other information

Table 2.1: Typologies of uncertainty in data. Adapted from the original table in Thomson et al., 2005.

MacEachren et al. (2005) also drew upon original research from Buttenfield (1991) to design a guideline on how to choose the best visual variables to represent uncertainty depending on the data type (e.g. points or continuous fields) and the data quality issue (e.g. positional accuracy or attribute accuracy). MacEachren et al. later (2012) reviewed these ideas by empirically testing and subsequently ranking several techniques according to their perceived intuitiveness for uncertainty visualization. Fig. 2.9 in subchapter 2.4 provides a summary of their conclusive findings.

Griethe & Schumann (2006) propose a classification of uncertainty visualization techniques based on the graphic characteristics of the uncertainty variate. They present four main types of visualization:

- Free graphical variables such as colour, size or focus, manipulated to show higher or lower uncertainty;
- Additional geometrical objects such as glyphs or grids superimposed on the main visualization;
- Animated and/or interactive representations, which are especially suitable for movement-like phenomena;
- Changes in rhythm, vibration and other non-visual approaches.

Senaratne & Geharz (2007) also produced an extensive classification of uncertainty visualization methods taking into account the data type and format, as well as the specific type of uncertainty and interaction between the user and the visualization (see Table 2.2). This was followed by the review of Kinkeldey et al. (2014), which elaborated the aforementioned previous findings into a new categorization that the authors summarized through the so-called *uncertainty cube*

(see Fig. 2.2). Their proposed classification divides techniques into four common dichotomies:

- Explicit vs. implicit, where data uncertainty is either represented directly through custom variables or signified indirectly, e.g. by showing multiple visualizations with several possible outcomes;
- Intrinsic vs. extrinsic, where an intrinsic approach manipulates existing symbologies (e.g. by altering colour or size) and an extrinsic approach adds new variables such as glyphs to the visualization;
- Coincident vs. adjacent, where uncertainty is either integrated into the original visualization along with the rest of the data or shown in a separate view;
- Static vs. dynamic, where a dynamic visualization uses interactive techniques to represent uncertainty.

Senaratne & Geharz (2007) also presented a fifth dichotomy, visually integral vs. separable, which, however, is seen as mostly overlapping with the intrinsic vs. extrinsic one.

Supported data type	Supported data format	Uncertainty type	Interaction type	Name of the method
	Raster, vector data	Attribute	Static	Exceedance probability mapping
				RGB colour scheme
			Interactive	Statistical dimension
		Attribute, positional	Dynamic	Animated isolines
			Static	Contouring

Continuous	Vector data	Attribute, positional	Static	Glyphs
	Raster data	Attribute, positional	Dynamic, interactive	Blinking pixels
	Raster, vector data	Attribute, positional	Dynamic	Animation
		Attribute	Dynamic, interactive	Blinking regions
			Static	Whitening
				Adjacent maps
				Symbol focus
				Opacity
Continuous, categorical	Vector data	Attribute	Static	Hierarchical spatial data structures

Table 2.2: Classification of uncertainty visualization techniques. Redrawn and adapted from the original figure in Senaratne & Geharz, 2007.

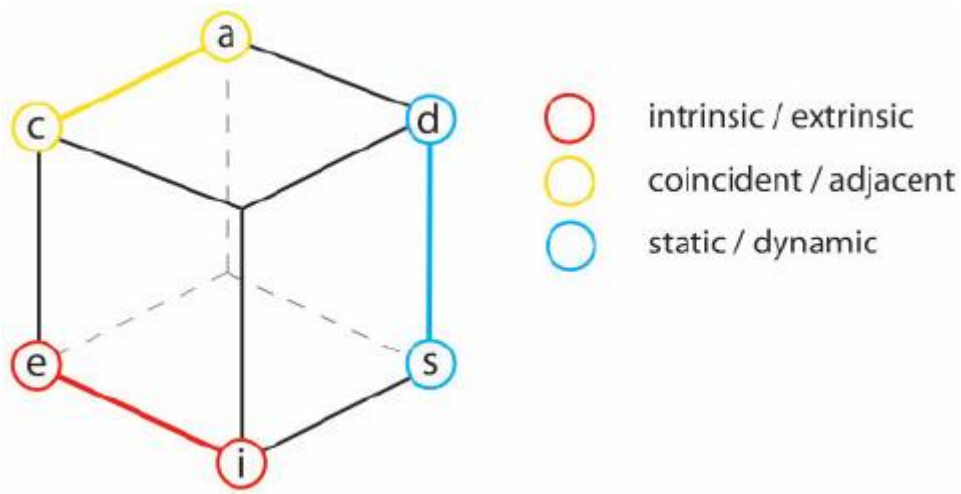


Figure 2.2: Uncertainty visualization cube (Kinkeldey et al., 2014).

When it comes to free graphical variables, Leitner & Buttenfield (2000) used texture, colour value and saturation to represent uncertainty. Unlike MacEachren (1992), they concluded that a lighter colour is more intuitively associated with uncertainty; the authors acknowledge, however, that this unexpected result may be due to the map being online-based instead of on paper. Kubíček & Šašinka (2011) later tested and contradicted Leitner's findings, thus confirming expected results. Pang (2001; see Fig. 2.3) showed how uncertainty can be intrinsically encoded in contour line gaps. Rhodes et al. (2003) experimented with both intrinsic and extrinsic visualizations, showing uncertainty through variations in hue, opacity and texture. Kunz et al. (2011) provided a summary of techniques used to visualize uncertainty in natural hazard maps, mostly focusing on graphical variables such as hue, value, saturation and transparency.

Additional objects to signify uncertainty were the focus of Beard and Buttenfield's (1999) review on how to visualize errors by including extrinsic glyphs in the map. The authors, citing research from Mitasova et al. (1995), argued that glyphs could be effectively used in uncertainty visualization through the alteration of parameters such as colour or height. Authors such as Wittenbrink et al. (1996) and Pang (2001; see Fig. 2.3) also showed examples of extrinsic uncertainty visualizations through additional glyphs, with the uncertainty itself mapped through variations in glyphs' width, length or orientation. Korporaal & Fabrikant (2019) opted instead to show uncertainty by overlaying a textured layer on the map (see Fig. 2.4).

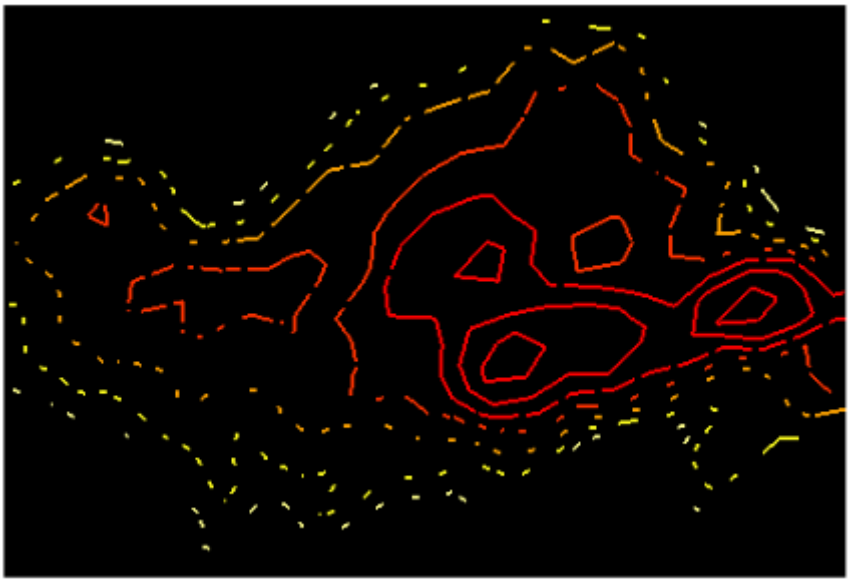


Figure 2.3: An example of intrinsic uncertainty visualization from Pang (2001). Gaps in contour lines signify high uncertainty.

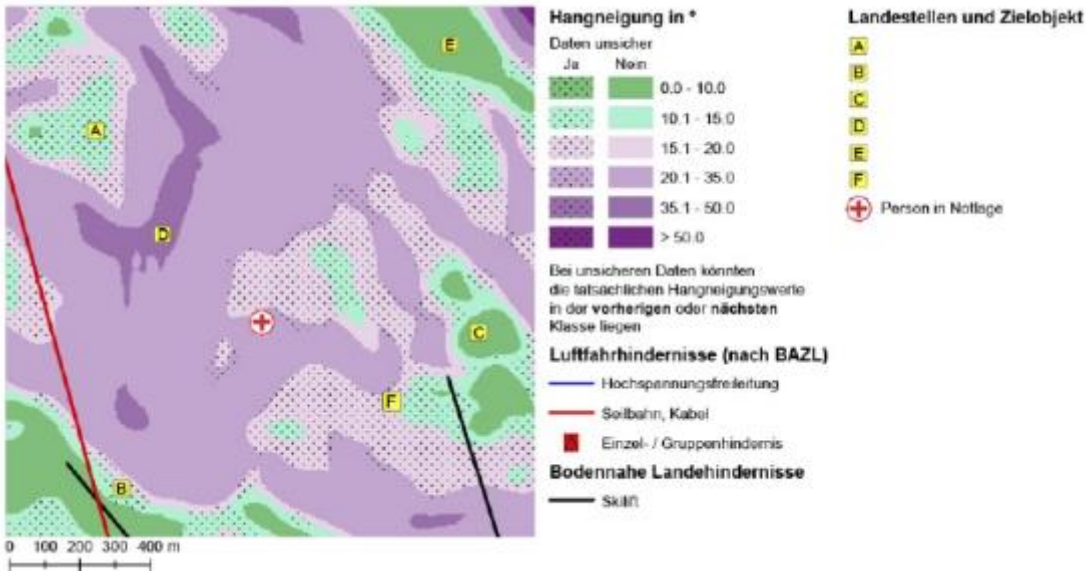


Figure 2.4: Terrain uncertainty map from Korporaal & Fabrikant (2019). The authors depict slopes with associated uncertainty visualized through a dotted texture layer.

Finally, Evans (1997), Lodha et al. (2002) and Grigoryan & Rheingans (2004) experimented with complex techniques and interactive animations to visualize uncertainty in variables such as movement or surface geometry.

This vast body of research provides the theoretical foundation for the uncertainty visualization techniques that will be used in the present thesis. However,

the aforementioned studies do not always take into account the complex interplay of uncertainty visualizations, heuristics and reasoning, which is the subject of further research.

2.3 Heuristics, biases and visualization

MacEachren (2015) and Kinkeldey et al. (2017) argue that the concept of reasoning under uncertainty is an under-researched issue in all spatial sciences.

The topic of reasoning under uncertainty is especially relevant in the broader context of visual biases, as people commonly employ heuristics when pondering outcomes and different visualizations play a significant role in affecting users' decisions (Deitrick, 2012; Reani et al., 2019). Visualizations may trigger both correct and incorrect associations between events and variables, therefore calling the availability heuristic into play. If they are poorly constructed, visualizations can also generate faulty interpretations of the depicted statistics, therefore leading to the aforementioned representativeness heuristic, e.g. by not correctly showing the randomness of a single event (Zuk et al., 2006). At the same time, visualization techniques have a high potential to mitigate these errors and produce better judgements (Zuk et al., 2006). Zuk & Carpendale (2007) argue that uncertainty visualizations should be consistently linked through continuous feedbacks with correspondent stages of reasoning, in order to allow cartographers to support decision-making by strategically improving and evolving these visualizations. The same authors also extended the uncertainty taxonomy from Thomson et al. (2005) to the process of reasoning under uncertainty (see Table 2.3).

Uncertainty category	*Reasoning definition*
Currency/timing	Temporal gap between assumptions and reasoning steps
Credibility	Heuristic accuracy and bias of analyst
Lineage	Conduit of assumptions, reasoning, revision and presentation
Subjectivity	Amount of private knowledge or heuristic utilized
Accuracy/error	Difference between heuristic and algorithm
Precision	Variability of heuristics and strategies
Consistency	Extent to which heuristic assessments agree
Interrelatedness	Heuristic and analyst independence
Completedness	Extent to which knowledge is complete

Table 2.3: Typologies of uncertainty in reasoning. Adapted after Zuk & Carpendale (2007)

It is therefore paramount to study how individuals perceive visualizations in general as well as, on a smaller scale, how they interpret the symbols used. Smallman & St. John (2005) have shown for instance that viewers tend to prefer more realistic visualizations as they intuitively perceive these visualizations to be more accurate, in a bias that the authors called *naïve realism* (see Fig. 2.6).

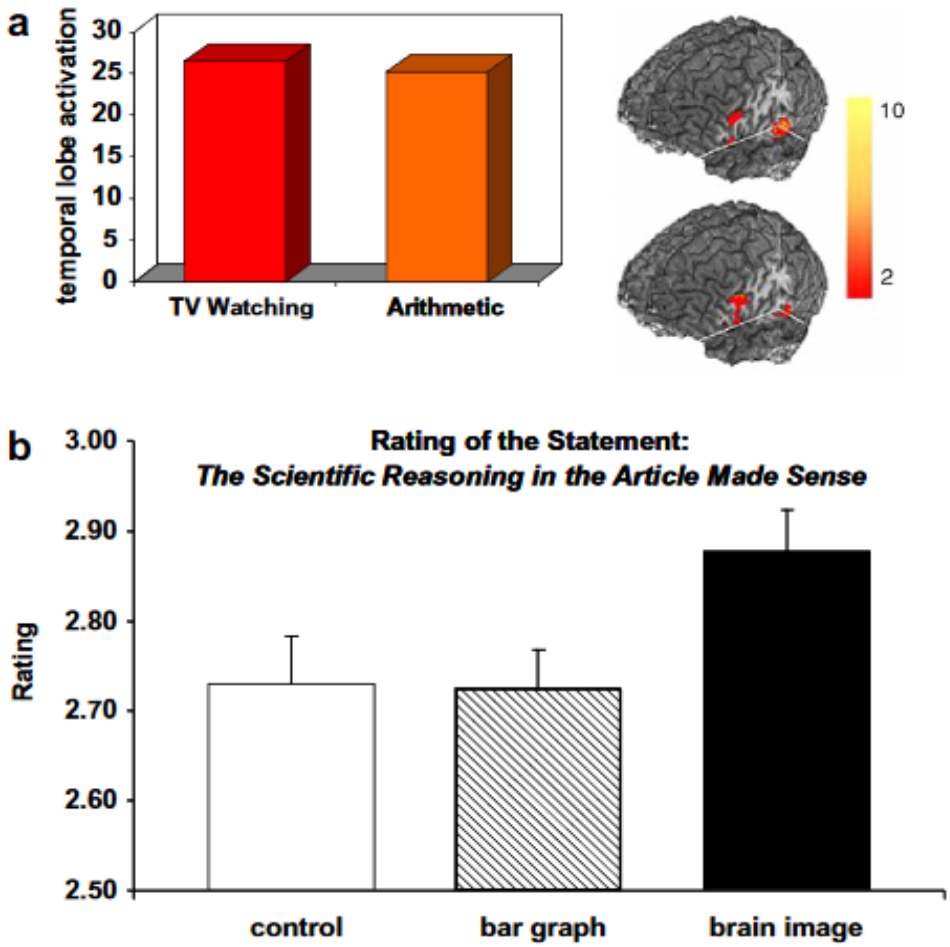


Figure 2.5: A comparison between bar charts and brain imaging. Users perceived the latter as significantly more accurate. (McCabe & Castel, 2008)

Biases can also be ambiguous and oppose each other: some viewers prefer familiar visualizations (*viewpoint inertia)*, while others might choose to seek novelty (Bailey et al., 2007). Users also often prefer high-quality imaging to other lower-quality visualizations, which are consequently perceived as less accurate and trustworthy regardless of their actual content (McCabe & Castel, 2008). The issue of high-quality imaging is of special importance in the context of medicine, where professionals rely heavily on imaging to take decisions and make judgements often within a short timeframe (Itri & Patel, 2018; Hughes & Dossett, 2020). Several studies (e.g. Mayr et al., 2019; or Xiong, 2019) have also found out that users generally perceive visualization as more accurate and, therefore, more trustworthy the more information it includes.

In the context of visualization perception and visual semiotics, it is important to note that the mere presence of a visualization can inherently produce biases (Padilla et al., 2018). Visualizations perceived to be more compelling to the eye can positively affect map usability and judgement accuracy (Fabrikant et al., 2010; Hegarty et al., 2010). However, the same visualizations could also cause users to overlook other information that might be more relevant to map reading or to the specific task they are being asked to solve (Stone et al., 1997; see Fig. 2.7).

Figure 2.6: An example of base rate fallacy applied to visualization. The difference between the two scenarios is statistically insignificant, but viewers are led to believe otherwise because they tend to focus on the visualization while ignoring the accompanying text. (Stone et al., 1997)

Symbols are also deeply interlinked with cognitive biases. Tversky (2011) describes symbols as " pictorial depictions of thought " that we use to organize and synthesize reality mentally; glyphs such as lines or bars are highly context-dependent and are associated with intuitive meanings; therefore, they may cause poor judgements when used counterintuitively in a visualization. For example, larger size is associated with higher relevance, lines are associated with a connection between two events and arrows are associated with causality (Tversky, 2011). Viewers best understand continuous metrics such as frequency when represented with congruent visual variables such as thickness, whereas the use of non-continuous metrics may lead to misinterpretations of the visualization (Tversky et al., 2011; see Fig. 2.8). Furthermore, abstract glyphs are often

derived from iconic symbols, which increases their visual significance (MacEachren et al., 2012).

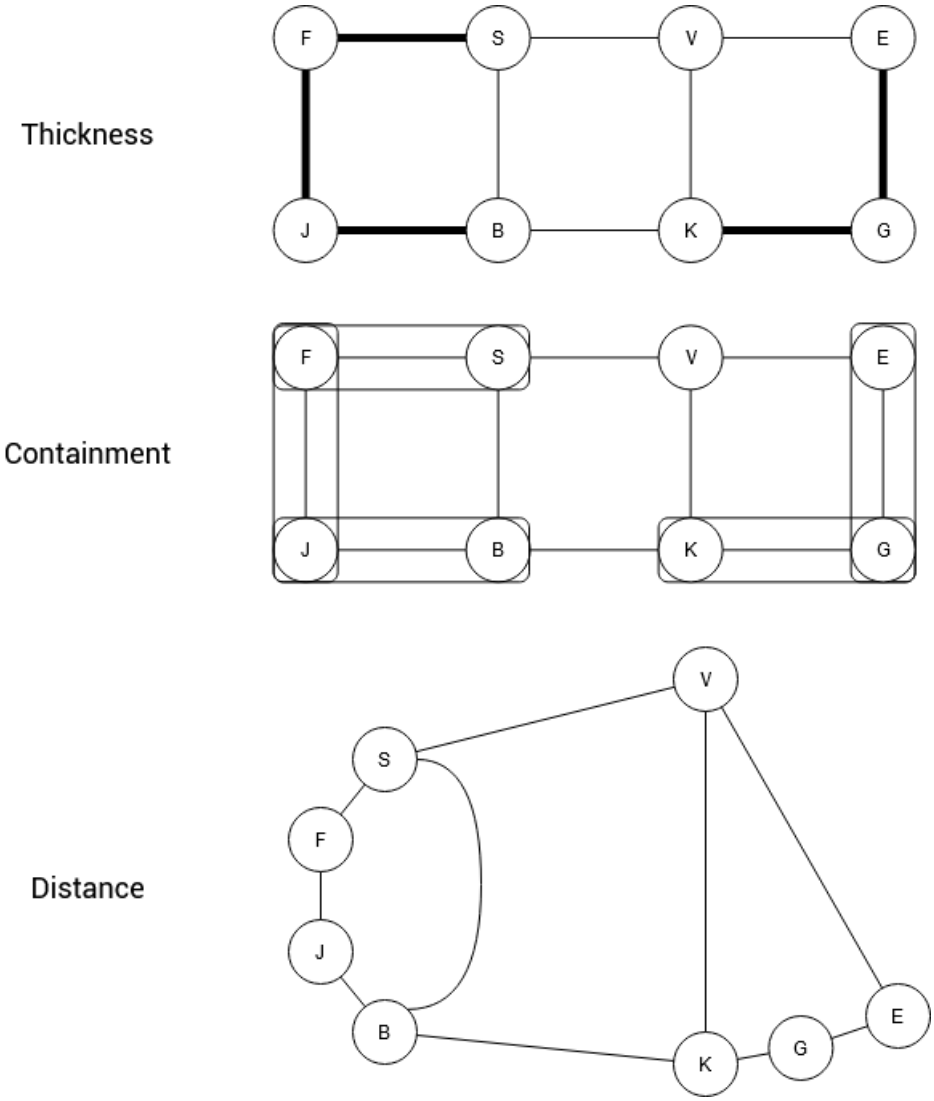


Figure 2.7: "Diagrammatic prompts". Tversky et al. (2011) introduced them to visualize relation frequency between nodes. The use of thickness and distance led to a significant improvement in user performances. (Figure redrawn and adapted after Tversky et al., 2011)

Therefore, each visual variable has a preferable use and a conventional interpretation that must be taken into account in the final visualization output. In the context of reasoning under uncertainty, MacEachren et al. (2012) have for example built an extensive guideline to best support users and map-makers alike by ranking visual variables according to their intuitiveness for uncertainty visualization (see Fig. 2.9).

Over time, cartographic science has absorbed these findings and has made use of them to improve map usability and experience and support viewer's performances.

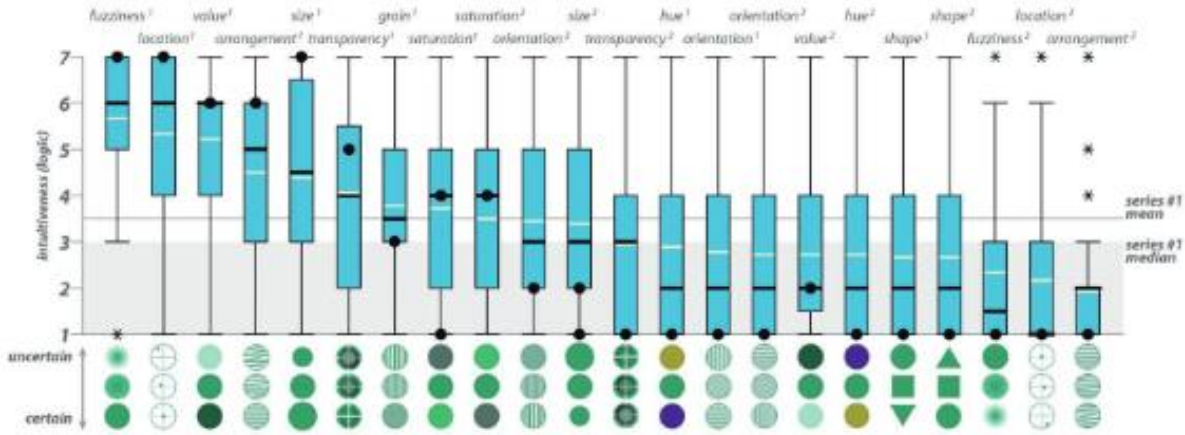


Figure 2.8: Visual variables ranked by their intuitiveness for uncertainty visualization. MacEachren et al., 2012

2.4 Geospatial visualization, reasoning and biases

Since at least the 1990s, research on how to visualize uncertain geospatial data has been aplenty (see e.g. MacEachren, 1992). However, as previously stated, studies about reasoning under geospatial uncertainty and related biases are uncommon. Several authors (e.g., Harrower, 2003; Kinkeldey et al., 2014; MacEachren, 2015; Chuprikova et al., 2018) have made calls for a deeper research focus in that direction.

With that said, there is a growing field of studies dealing with the effects of different uncertainty visualizations on decision-making, i.e. both on the *outcome* of the decision and on the *process* leading to it, regardless of what the actual outcome is (e.g., Harrower, 2003; or Kübler et al., 2019).

Research has focused on metrics such as correctness and timing to explore the relationship between uncertainty visualizations and geospatial decision-making; its results have been somewhat ambiguous at times. A study by Leitner & Buttenfield (2000) showed that the inclusion of uncertainty information in maps can aid users' decision-making, and this effect can be controlled through the

choice of certain visual variables such as colour value, hue or saturation; however, the conclusions concerning the variables themselves were not fully consistent with the available literature. Later work by Hope & Hunter (2007) further expanded these findings, highlighting that the effect of uncertainty visualization on users' response to maps can be linked to biases and heuristics known from the existing literature. More specifically, users were shown a map of land zones with different levels of suitability for a hypothetical airport; the suitability value was associated with a degree of uncertainty (see Fig. 2.10). When asked which zones they would personally prefer to build the airport, they would display a choice pattern consistent with known biases such as loss aversion (Tversky & Kahneman, 1979) and ambiguity aversion (Ellsberg, 2001).

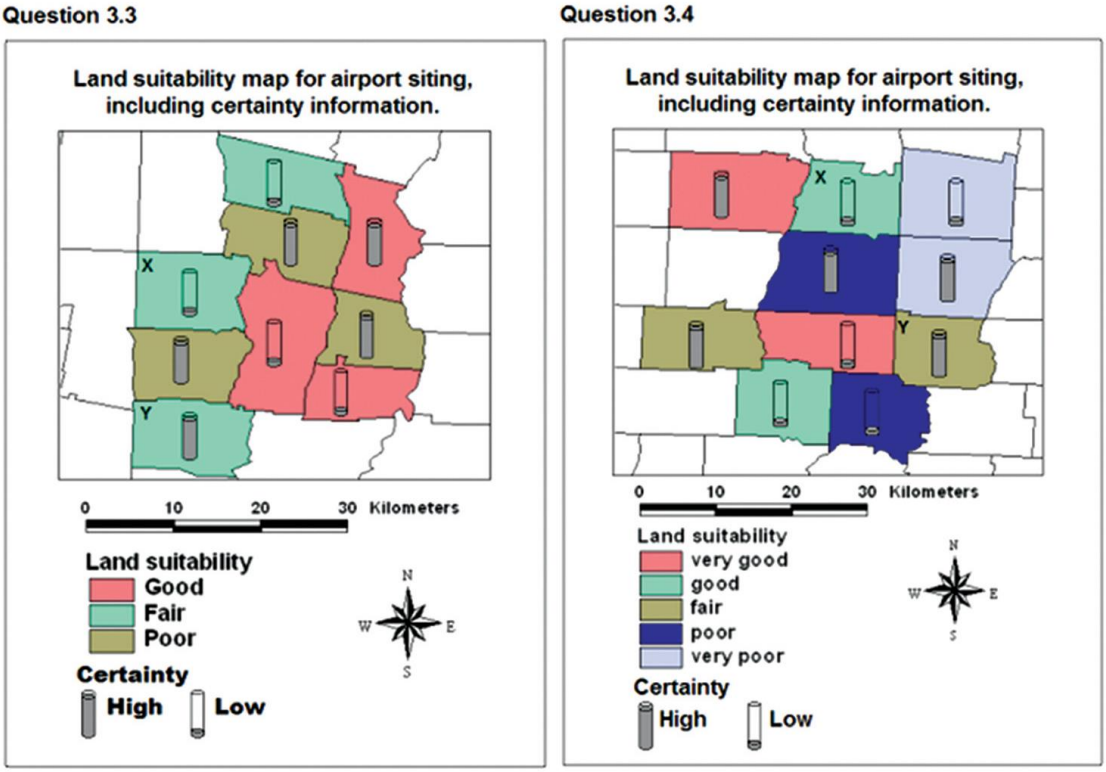


Figure 2.9: Study from Hope & Hunter (2007) on the effect of visualization of uncertainty on decision making. The two maps show several adjacent zones with different suitability levels for airport siting, along with information about data uncertainty.

However, Hope & Hunter (2007) acknowledged that these biases might lead to irrational decisions. Indeed, it has been noted that the choice of certain visuali-

zation through different salient (i.e. attention-inducing) map elements can trigger *visuo-spatial biases* (Padilla et al., 2018) that in turn only affect viewers' performance positively when they are conductive to a correct map interpretation (Padilla et al., 2018). Kübler et al. (2017) have found that, in the presence of natural hazard maps, the inclusion of uncertainty visualization could lead to riskier decisions than a map without any uncertainty visualization, thus contradicting previous research (e.g., Leitner & Buttenfield, 2000). The authors confirmed earlier findings (e.g., Hegarty et al., 2010) by underlining that the effects of uncertainty visualization might depend on several situational factors, as well as the users' background and personal knowledge and skills; therefore, no single result can be expected. Other works (e.g., Fabrikant & Skupin, 2005; Hegarty, 2011) have also stressed the need for the development of a "cognitively plausible" set of visual symbols for the uncertainty that best matches users' intuitions.

A study from Wilkening & Fabrikant (2011), where the authors asked map viewers to identify the best spots to land a helicopter on rugged terrain with potentially uncertain slopes, has focused on the complex relation between uncertainty visualization and reasoning under time constraints. While, as expected, more uncertainty information led to improved users' confidence and accuracy, the study found users performed best under moderate time constraints. The authors recognized this as a *speed-accuracy trade-off* bias; in other words, increased time pressure negatively affected the accuracy of users' choices. Further work from Korporaal & Fabrikant (2019) has shed new light on the issue, concluding that uncertainty visualization under time constraints may not necessarily affect outcomes (i.e. the final decisions). However, it does play a substantial role in the *processes* that lead to the final decision. Their findings further showed once again how uncertainty visualization, as well as its relation with known cognitive biases, has varied and sometimes unpredictable effects on users. While the outcomes in the study were consistent with a loss aversion bias (see Tversky & Kahneman, 1979), uncertainty visualization apparently *increased* the time it took for users to make decisions.

A relevant issue within the broader topic of reasoning under uncertainty is the *containment heuristic*, i.e. the tendency to intuitively consider what lies inside a border as thematically distinct from what lies outside (Padilla et al., 2018). Discrete boundaries are generally associated with a change in information and semantic meaning (Fabrikant & Skupin, 2005). The inclusion of discrete boundaries into the visualization can generate *deterministic construal errors*, as they can lead users to disregard the probabilistic meaning of the boundary by interpreting it as a deterministic border (Joslyn & LeClerc, 2013). McKenzie et al. (2016), in a study about users' interpretation of positional uncertainty, indeed found out that map users are more likely to adopt a containment heuristic when the probability distribution (uncertainty) of the location position is visualized with a hard border rather than with a fuzzy border (see Fig. 2.11).

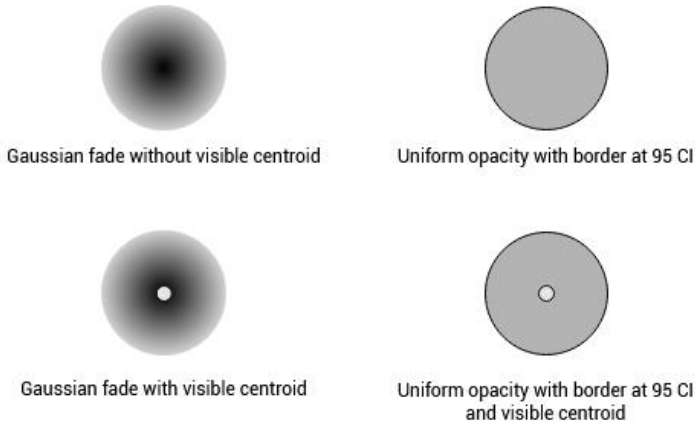


Figure 2.10: Four representations adopted for positional uncertainty (McKenzie et al., 2016). Uniform opacity was more likely to trigger a containment heuristic in map viewers. (Figure redrawn after McKenzie et al., 2016)

Drawing upon these findings, there have been several studies aiming to explore how to test, evaluate and mitigate the containment bias. A particularly useful resource has been the *hurricane tracks*, also known as the cone of uncertainty. The National Hurricane Center of the NOAA (National Oceanic and Atmospheric Administration of the United States) uses the cone of uncertainty as a tool to visually communicate the forecast for hurricane tracks, along with associated

probability levels. In this visualization, a series of points often connected through a centerline shows the predicted hurricane track, whilst a surrounding white-coloured cone defines the area where two-thirds of the storm paths have fallen into within the last five years (see Fig. 2.12). In other words, the cone only visualizes a probability distribution, without any information on the storm intensity or size as well as the likelihood of any specific path. Nor does it rule out the chances of a storm hitting areas outside the cone; in fact, one-third of hurricanes does exactly that.

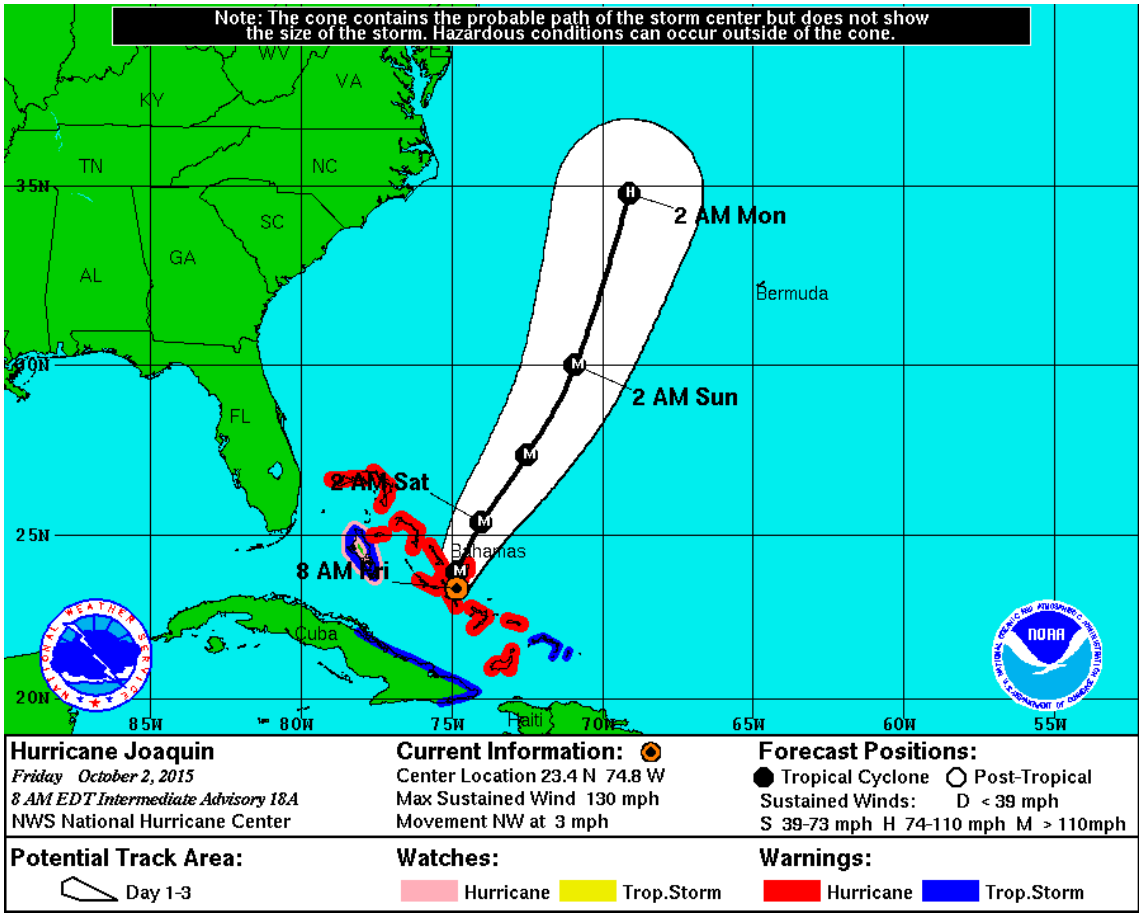


Figure 2.11: Cone of uncertainty for Hurricane Joaquin. National Hurricane Center, NOAA, 2015

Cox et al. (2013), among others, have found that the cone of uncertainty is commonly misinterpreted. They report how viewers inside the predicted cone overestimate the likelihood of being hit by the storm, whereas those living outside mistakenly believe themselves to be safe. This indicates that map readers equate the cone boundaries with the predicted storm limits, thus hinting to a

deterministic construal error (Joslyn & Le Clerc, 2013). The authors also proposed an ensemble view to mitigating these biases, with several hurricane tracks shown together to create regions with different path density instead of a single probabilistic cone.

Ruginski et al. (2016) have empirically tested their displays, along with a few others, and concluded that users are less likely to adopt heuristics such as containment or distance in the presence of ensemble views (see image C in Fig. 2.13). Therefore, they also tend to judge storm-associated risks more accurately. Padilla et al. (2017) later reviewed and confirmed such findings, suggesting that ensemble views are indeed helpful to control for biases related to size and containment which seem highly difficult to avoid under the standard summary display.

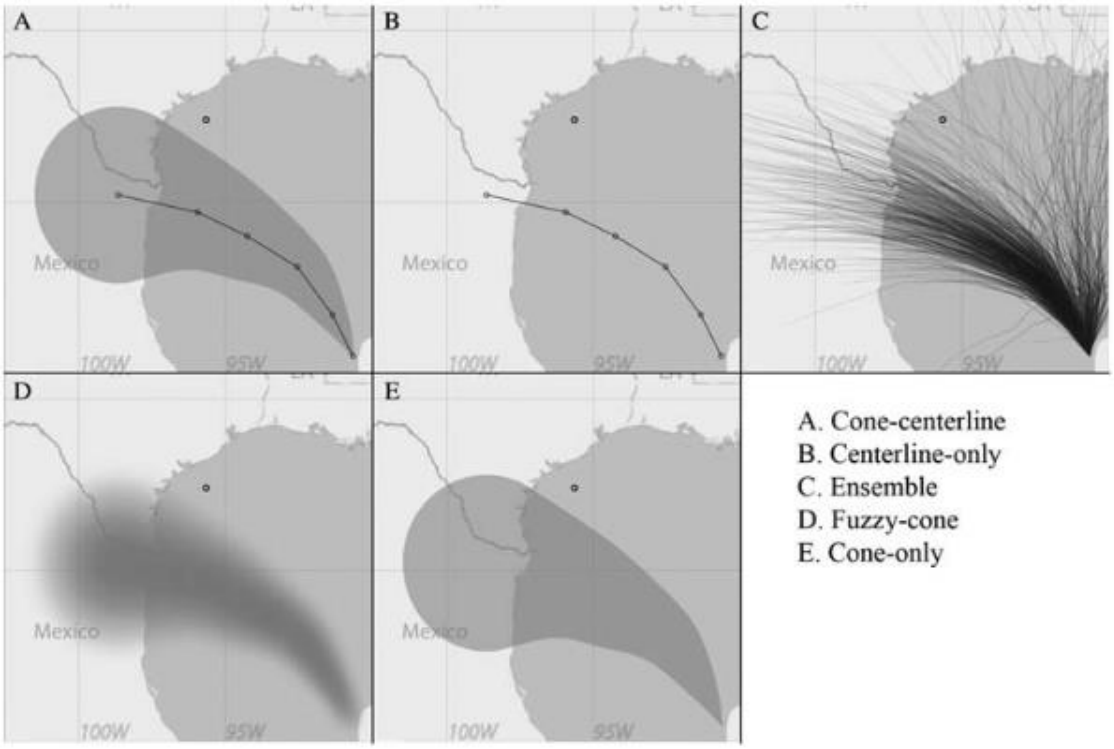


Figure 2.12: The five views tested by Ruginski et al. (2016).

The findings from Cox et al. (2013), Ruginski et al. (2016) and Padilla et al. (2017) are especially relevant, as they will serve as fundamental ground for this present research.

2.5 Summary

This literature review has outlined the state-of-the-art concerning heuristics and biases in visualization, with a special focus on the visualization of geospatial data.

Subchapters 2.1 and 2.2 have provided a theoretical framework on the role of biases and heuristics in reasoning and logical thinking. Findings across a diverse spectrum of knowledge domains have highlighted that the human brain makes heavy use of subjective mental depictions and thought patterns to make sense of reality. These images and patterns, while time-efficient and generally convenient, do not necessarily produce accurate outcomes in terms of judgements; nonetheless, they do form the basis of our decision-making process, especially under uncertain circumstances. Starting from the landmark works of Tversky and Kahneman in the 1970s, the so-called *heuristics* – logical shortcuts that help us filter out unneeded information and make quicker judgements – have become a central issue in cognitive science. The subchapters further listed a few commonly researched types of heuristics and made examples of their relevance in other fields as well, such as medicine.

Subchapter 2.3 has categorized general visualization techniques for uncertain data, following its most commonly accepted classifications and presenting an essential outline for each broad method type. Early research on visual variables from Bertin and, later on, academics such has MacEachren has initiated an extremely prolific body of research on the issue of uncertainty visualization. Authors have experimented with a vast number of techniques and variables, showing that no single suitable techniques exist. On the contrary, it is crucial to take into account the data type and format, as well as the uncertainty type and the desired visualization outcome, in order to produce a meaningful cartographic result.

Finally, subchapters 2.4 and 2.5 have dealt more specifically with biases and heuristics in visualization and reasoning under geospatial uncertainty. It has been highlighted that users tend to have a heuristic-driven approach to different uncertainty visualizations, which all have intuitive meanings associated to

them. As a result, the choice of a visualization technique can considerably affect map perception and map-related judgements and decisions. As research has shown (e.g., Leitner & Buttenfield, 2000; Hegarty, 2011; or Korporaal & Fabrikant, 2019), uncertainty visualization can help us to shed further light on the topic of visual semiotics and visuo-spatial biases – i.e., how users perceive and interpret map symbols and how they reason through them.

The heuristics of containment and distance were the subject of additional focus. Research has shown that boundaries in maps act as a powerful semantic divide: in other words, users perceive what lies inside a bounded region as different from what lies outside. Furthermore, this perceived difference may increase the further away an object lies from the said boundary. Map makers can therefore manipulate border visualizations to affect users' perception and data interpretation, up to the point of conveying misleading or even outright false information. Thus, the problem becomes all the more relevant in the presence of uncertain data, such as unknown spatial locations. The present work aims to provide new findings on this issue by testing several boundary-related visualization techniques for uncertain data and their effects on users' reasoning, choices and map perception.

3. Methodology and case studies

3.1 Heuristics: what and how

Due to their somewhat subjective nature, heuristics are challenging to study using statistical analyses. In fact, while there are several definitions available for heuristics, there are no established methodological principles to investigate their use and formal models to explain heuristics may not be developed enough to provide a solid scientific framework for empirical studies (Gigerenzer & Gaissmaier, 2011). The issue of *how* to detect heuristics in user studies remains challenging; one common attempt to do so was to first identify errors in users' judgements and then use heuristics as potential explanations for such errors (Gigerenzer & Gaissmaier, 2011). However, authors have used a diverse set of approaches to this research subdomain, from regression analyses (e.g., Kübler et al., 2019) to fully qualitative methods based on users' explicit statements about their logical processes (McKenzie et al., 2016).

The present thesis adopted a mixed approach. In order to measure and interpret the potential use of the heuristics identified through the literature review as described in Chapter 2, users' responses to maps were first coded numerically to detect any significant differences. If a pattern consistent with a relevant heuristic arose from the analysis, it was tentatively regarded as evidence of heuristic use when no other explanation was immediately available. Furthermore, the investigation also included open-ended statements that served as additional tools to support the overall reliability of the conclusions by either confirming or casting doubt on numerical findings. For instance, as border visualizations were the main subject of this study, numerical differences between users' responses from different maps were considered evidence of a containment heuristic if these changes could only be explained by changes in the visualization driving the heuristic itself. In other words, statistical changes in map perception could arguably be sufficient evidence of a containment heuristic if the only (or main)

difference between the maps was the visualization of the border, and subsequently the potential semantic meaning of the area inside it.

Similarly, different perceptions for different locations could be evidence of a distance heuristic if the distance from a border was the only piece of information available to tell the locations apart. Open-ended statements could later confirm these findings if users had explicitly referred to one of the relevant heuristics as a driver for their reasoning process. The same mixed approach could serve to suggest the use of other heuristics cited in the literature, if numerical patterns were consistent across at least two maps and, once again, no other immediate explanation for such patterns was available.

As previously mentioned, it is challenging to detect heuristics by using objective measures exclusively. Some subjective measures, both through users' personal claims and through the qualitative interpretation of the results, were necessary to extend initial findings and come up with meaningful conclusions. However, the numerical analysis provided a reliable and reproducible framework. The following subchapters describe the methodological approach in greater detail.

3.2 Map design and case studies

Identification and general workflow

The main goal of the present thesis is to investigate the links between heuristics and visualization of geospatial uncertainty and, consequently, discover how heuristics affect users' perceptions of certain visualization methods. To do so, the premises mentioned in Chapter 2 have acted as an inspiration to lay out the following methodological workflow:

- Identification of study cases,
- Conceptualization and design of the relevant maps, and
- Design and administration of a user survey in the form of an online questionnaire, including a pre-test.

The first stage has brought about the selection of two case studies of natural hazards deemed as potentially relevant for the scope of the project. The choice

of two case studies reflects the dichotomy between the *visualization of uncertainty* – which refers to the uncertainty already present in the original data – and *uncertainty of visualization*, that takes into account all the inaccuracies potentially arising from the data analysis process (Brodlie et al., 2012). Additionally, while the issues of containment and distances were the subject of several studies from different perspectives (see e.g., Newman & Scholl, 2012; or Grounds et al., 2017), the most relevant findings for geospatial data have come from case studies with natural hazards, as seen in the aforementioned studies of Ruginski et al. (2016) and Padilla et al. (2017). Natural hazards and general terrain-related risks have also been the subjects of studies by Leitner and Buttenfield (2000) or Korporaal & Fabrikant (2019). Therefore, they were also selected as the main background for the present work, as they have clearly emerged as powerful case studies to evaluate the effects of uncertainty visualizations and more specifically visualizations of boundaries.

The first case study is air pollution and, more specifically, the diverse territorial distribution of the air pollutant PM10, whereas the second case is an avalanche risk map. In the former, uncertainty arose from the analysis of point data, while in the latter some uncertainty was inherently present in the original dataset.

Both cases share a similar conceptualization. The core cartographic product of this work was the creation of several maps, using the ESRI software ArcMap 10.8, to visualize the distribution of these two natural hazards across an area of interest. In these maps, boundaries between different risk classes were visualized using different methods, e.g., a gradient or a hard border, in a process that will be here called "Borderization".

To better evaluate users' perceptions about such methods, the following step was to superimpose several potential housing locations across different risk classes on each map and then ask users to rate them accordingly to their perceived level of desirability and safety. Kübler et al. (2019) had already experimented with the use of potential housing locations to evaluate uncertainty visualizations; in the present work, the locations and their assigned ratings acted as primary tools to detect possible patterns in heuristics' use. Table 3.1 shows

general initial assumptions on how the different heuristics were expected to manifest themselves through users' behaviours and choices in the survey. The next subchapters will present the two case studies and the associated questionnaire in greater detail.

Heuristic	Assumption(s)
Distance Containment	• Significant difference in perception between two locations in the same thematical category but differing in their distance from a thematical border • Significant impact of borderization change on differences in perceptions between two map locations • Significant perceptive alterations after the introduction of extrinsic uncertainty
Availability	• Reported difference in perception between maps depicting different case studies due to lived experience
Adjustment to anchor	• Repeated perception patterns and user behaviours across successive maps
Representativeness	• Significant preference for certain colours to indicate high-risk levels • Significant preference for certain borderizations to represent a specific natural hazard

Table 3.1: General initial assumptions on the relation between heuristics and users' behaviour in the survey.

Case study I: Data collection and pre-processing

The WHO (World Health Organization) defines PM, or particulate matter, as a "widespread air pollutant, consisting of a mixture and solid and liquid particles suspended in the air" (WHO Europe, 2013, p. 3). The name PM10 refers to particles with a diameter of less than 10 μm; common sources for these pollutants are anthropogenic activities such as combustion engines, industry and road traffic (WHO Europe, 2013, p. 3). Owing to their extremely small size, pollutant particles can penetrate inside the human respiratory system and potentially cause a number of severe ailments, including asthma, lung cancer, and stroke (WHO Europe, 2013, p. 6). Dense concentrations of PM10 are a known cause of excess mortality in the affected regions (EEA, 2019).

PM10 was a relevant metric for the scope of this study as it is a widespread natural hazard that most people experience daily, yet it is rarely visible and it does not necessarily have precise spatial boundaries. Therefore, the choice of a specific visualization to show PM10 "borders" is not trivial.

PM10 concentration data are publicly available in the EEA (European Environment Agency) website under various data formats. The agency also provides a

dataset made of three distinct shapefiles with georeferenced point data about PM10 concentrations in the year 2017, from 2491 different measurement stations across Europe. Each shapefile contains a specific subset of stations: urban background (1384 stations), rural background (361 stations) and traffic (746 stations). The shapefiles include data about the average yearly PM10 concentration and the 90th percentile of the daily value distribution. For this study, urban background stations were numerous enough to guarantee sufficient data accuracy across the whole continent, unlike the rural background stations; they also did not show the extreme peak values that the traffic stations sometimes showed. At the same time, the varying geographical distribution of urban background stations over Europe also guaranteed a degree of uncertainty that was relevant for the study. Therefore, the subsequent analysis focused only on the urban background stations.

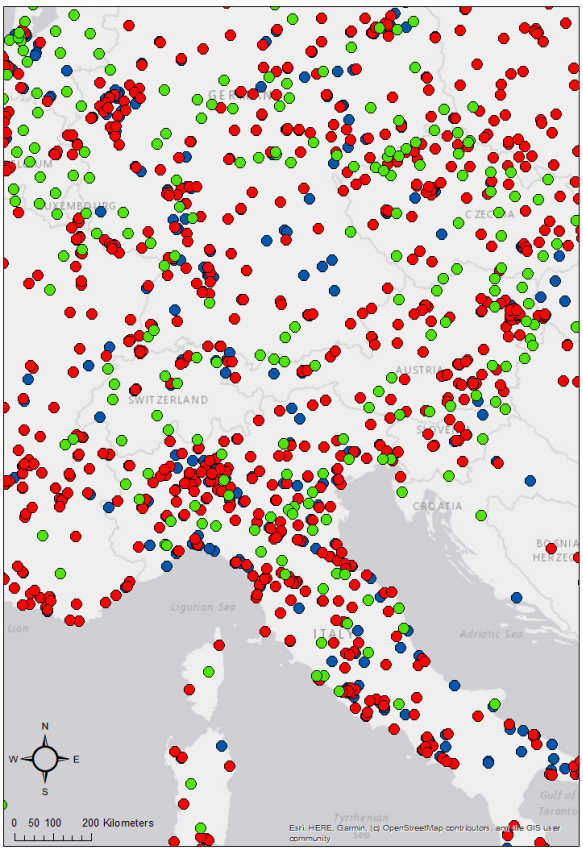
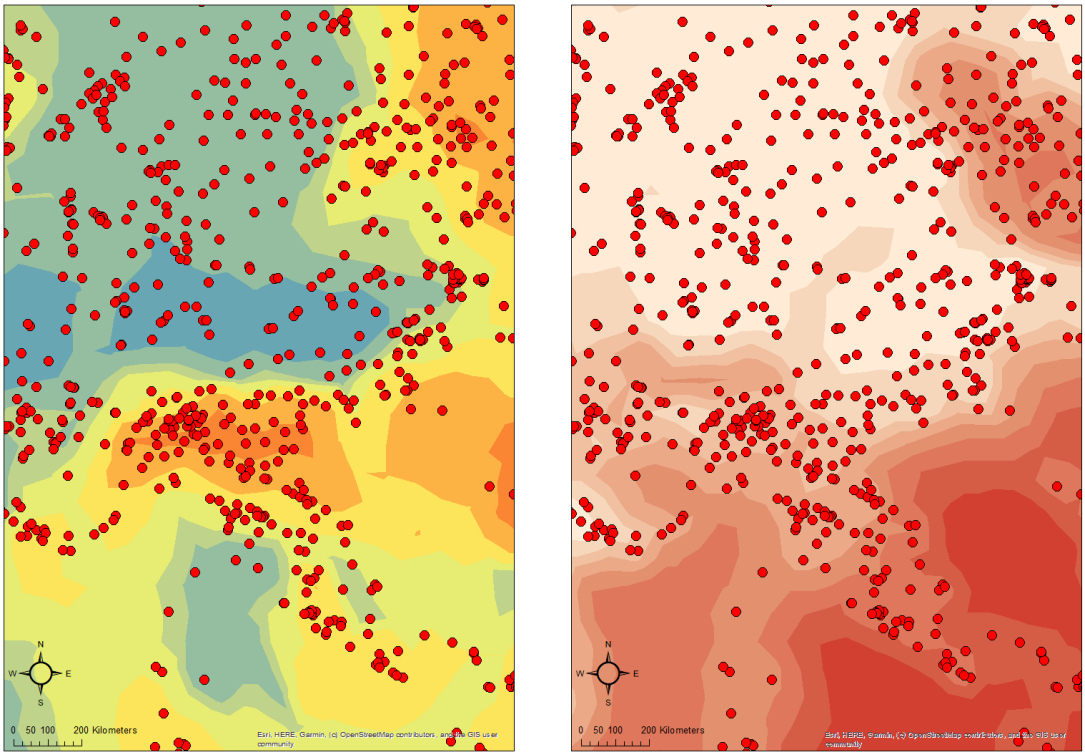


Figure 3.1: A sample of stations from the EEA dataset. Green dots indicate rural background stations and blue dots are traffic stations. Urban background stations, here represented as red dots, formed the basis of this study's analysis.

To turn point data into a continuous field, the software ArcMap offers several options for Kriging interpolation. Kriging is an interpolation technique that makes use of a semivariogram to predict and plot values across space from a set of known points by estimating their spatial codependence (Krivoruchko, 2012). Among different types of Kriging, ArcMap has an in-built function for Empirical Bayesian Kriging, which is a Kriging method that estimates several successive semivariogram models to account for the potential error of a single semivariogram (Krivoruchko, 2012). This technique proved especially useful for this present thesis as it provided spatial error estimates, which could act as proxies for data uncertainty. In fact, Kriging error measures how likely Kriging estimates are to be valid on a certain location (i.e., how likely the interpolation is to be accurate) and it depends on the amount of input data used for the interpolation as well as on local geographical factors (e.g., whether there are large class variations within a narrow space). Therefore, Kriging and the associated error were the basis for the successive map creation (See Fig. 3.2a, b).



a *b*

Figure 3.2: Initial results of Empirical Bayesian Kriging on PM10 average concentration (a) with associated standard error (b). Blue and lighter areas indicate lower PM10 values and lower errors respectively.

Case study I: Map design

After performing the interpolation, the next step is selecting the area of study. Northwestern Italy is especially relevant, as a map of this area allows users to detect significant variations in PM10 concentration within a small space – from the high values around Milan and Turin to the low values of the Alps and Southern Switzerland. As the original data points have an uneven distribution, with much higher density in the Po plains compared to the Alps, Kriging interpolation in this area also provides several spots of relatively high error that were useful for visualizing data uncertainty.

Kriging results were then "borderized" in four different ways by manipulating different visual variables. The first visualization shows a region with high PM10 concentration, coloured in red and delimited by a single line as a hard border; values outside the area have no assigned class, therefore the space in the map is dichotomic (Fig. 3.3a). The high PM10 concentration region is the area with yearly averages of PM10 daily concentrations exceeding 20 μg/m³, the threshold that WHO defines in its air quality guidelines as overall safe for human health (WHO, 2005). The second visualization shows the same space and data with three different classes instead of a dichotomy; this classification divides the area into high, moderate and low concentration, each having a hard border, with class colours from red for high to light pink for low concentration (Fig. 3.3b).

The last two maps visualize the same data as the first two, but with a gradient border instead of a hard one. In the third map, the high concentration area has a gradient border smoothly transitioning from red to white (Fig. 3.4a), while the fourth map shows a red-to-orange gradient across the entire space from high to low concentrations (Fig. 3.4b). Echoing the classification of visual variables described by MacEachren et al. (2012), different colour values represent different data classes in the first two maps, whereas the manipulation of the contour fuzziness in the last two maps serves to simulate the transition from high to low PM10 values.

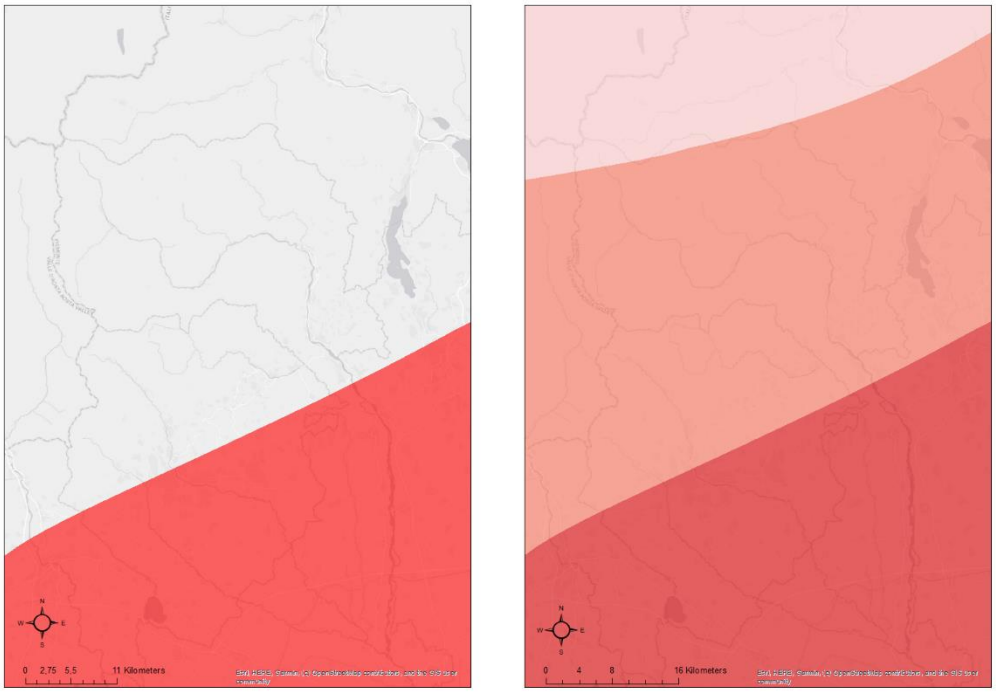


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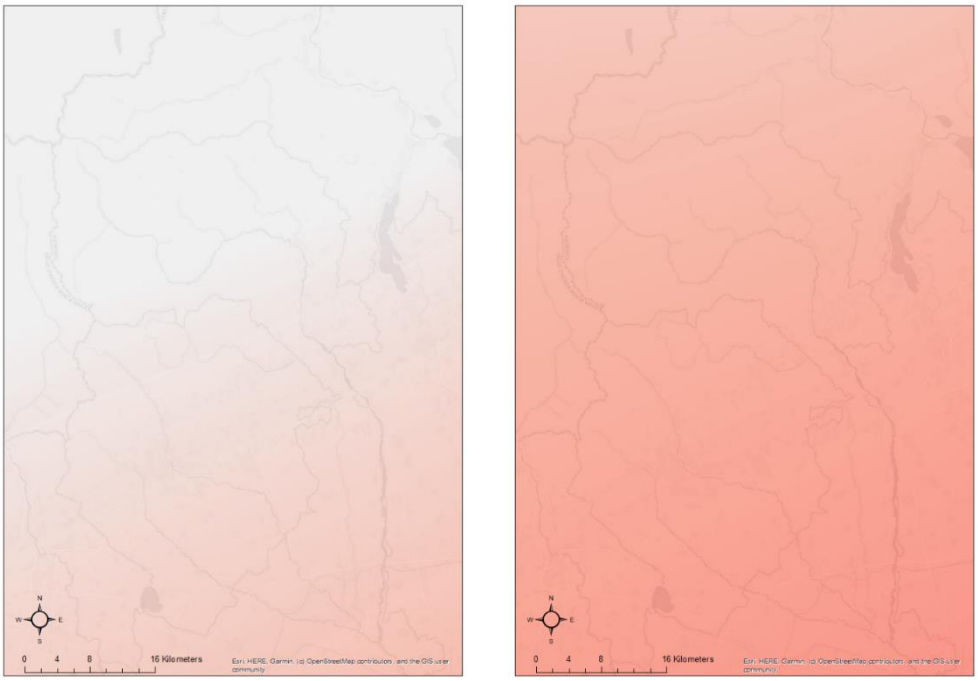
Figure 3.3: Details of the first two borderizations. a: Single Hard Border. b: Layered Hard Border.

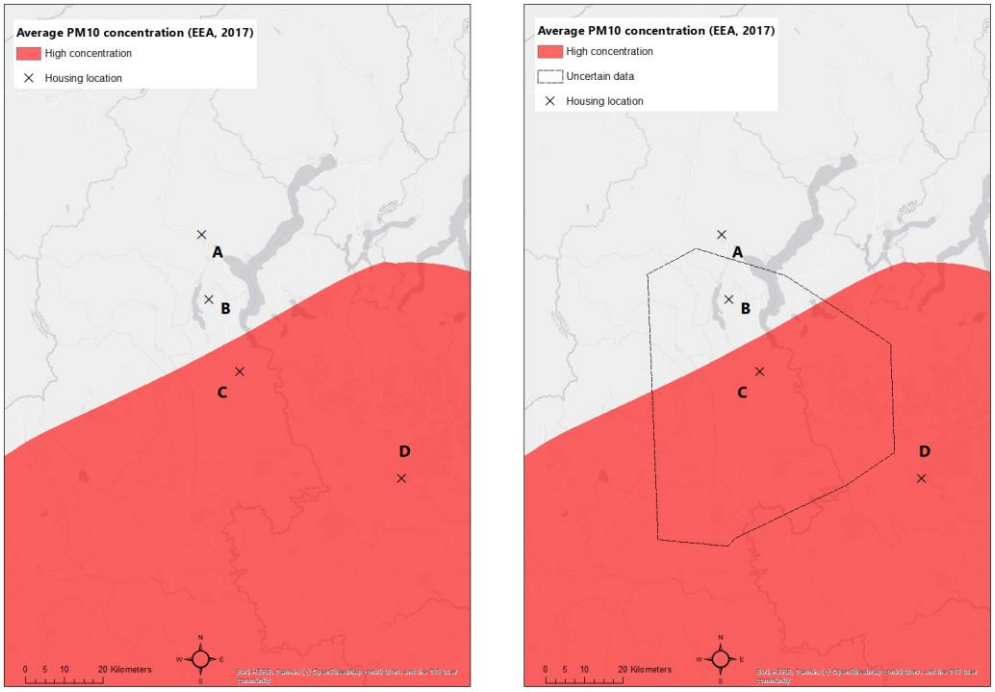


a *b*

Figure 3.4: Details of the last two borderizations. a: Limited Fuzzy Border. b: Total Fuzzy Border.

Each map has a copy that includes an area of uncertain data, defined as the upper half of Kriging error values. The area is visualized as a polygon over the rest of the map, with a thin dashed line as a border. This visualization choice is meant to potentially introduce further reasoning linked to borders and change perceptions among users looking at the maps.

All the maps also include four potential housing locations, two of which are located in the high concentration area. Two locations lie within the uncertain data area, with one of them also lying in the high concentration area (Fig. 3.5-3.8). Subchapter 3.4 describes how the final questionnaire makes use of these housing locations.



a *b*

Figure 3.5: Single Hard Border maps. Map in Figure b (right) includes extrinsic uncertainty.

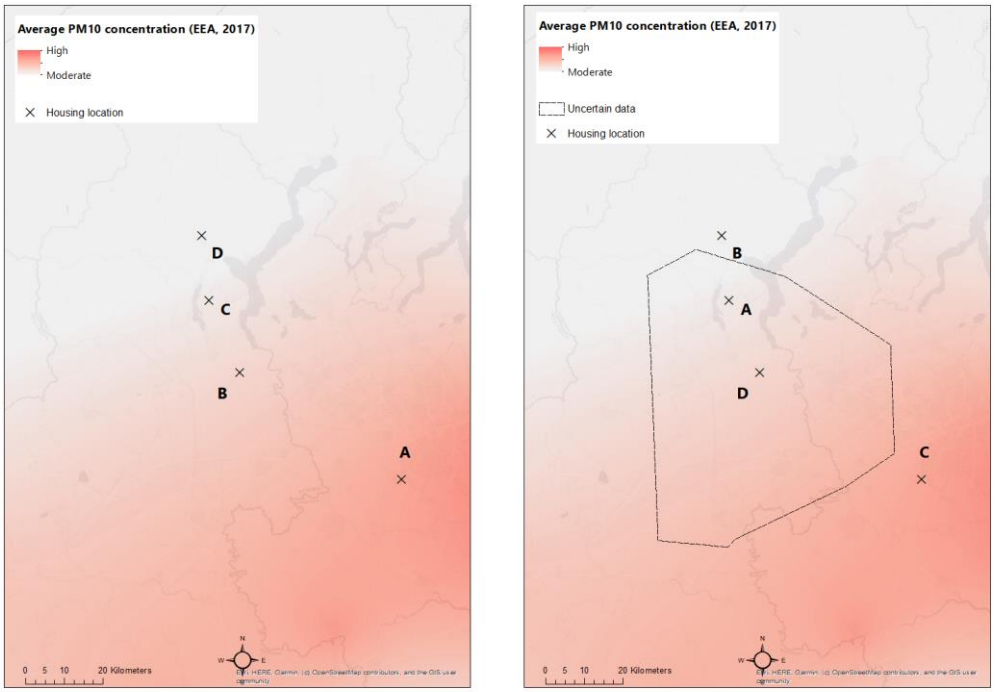


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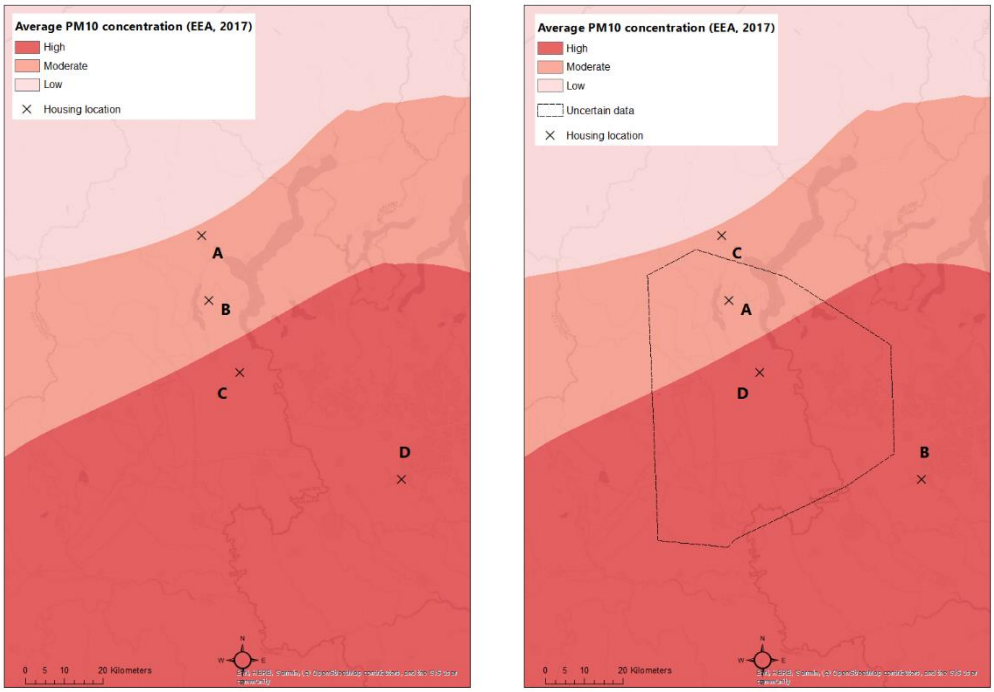
Figure 3.6: Limited Fuzzy Border maps. Map in Figure b (right) includes extrinsic uncertainty. Housing locations are randomized for reasons explained in Subchapter 3.4.



a *b*

Figure 3.7: Layered Hard Border maps. Map in Figure b (right) includes extrinsic uncertainty. Housing locations are randomized for reasons explained in Subchapter 3.4.

a *b*

Figure 3.8: Total Fuzzy Border maps. Map in Figure b (right) includes extrinsic uncertainty. Housing locations are randomized for reasons explained in Subchapter 3.4.

Case study II: Data collection and pre-processing

The second case study is a visualization of avalanche risk; this type of hazard is relevant for this thesis, as avalanches are a "moving" hazard where factors like distance and boundaries play a significant role in influencing users' perceptions and decisions.

The goal was to obtain spatial data with some potential positional inaccuracy included, in order to be able to visualize uncertainty information in the final maps. A solution to do so was to make use of the *Naturgefahren-Hinweiskarten* (eng. "Indicative maps for natural hazards"), which are a particular type of natural hazard maps commonly produced by several geological and geoinformation offices across Germany, Austria and Switzerland. In contrast to the usual *Naturgefahrenkarten* ("Natural hazard maps"), *Hinweiskarten* have a relatively low degree of detail and spatial accuracy and only provide a general overview of the areas where extreme natural events are more likely to happen. They do not

usually include information about the potential intensity of such events (BAFU-Swiss Environmental Agency, 2020).

Many Swiss cantons provide large open datasets of natural hazard maps for public use. Among these, the geospatial databank of the canton of Valais-Wallis is one of the most extensive and detailed. It is possible to download a shapefile containing a *Hinweiskarte* with areas at risk of water- and avalanche-related natural events. The latter ones were the main subject of the second study case (Fig. 3.9).

However, the metadata did not provide additional information on the exact positional uncertainty of the boundaries of these areas. As many areas also show large differences in slope and surface typology, it may be that there is no single positional uncertainty value across the whole map and some boundaries might be more precise than others. For this study, these boundaries received an arbitrary positional uncertainty value of 100 meters in both directions.

Case Study II: Map design

After data collection, the following step is to select the study area. The valley of Saas Grund, in the southeastern part of the canton, seemed appropriate as the high risk area is particularly close to roads and human settlements, thus potentially increasing the visual perception of a threat.

The logical workflow to map the high risk area is the same as in the previous case study, although visual variables are altered differently to achieve the "borderization" results. The first "borderization" is a gradient border on a red-to-white scale, with a 100-meter width to show positional uncertainty as discussed before (Fig. 3.10). Unlike the PM10 case study, however, only the 100-m buffer has this gradient, whereas the high-risk area is completely coloured in red.

A layered border with several different risk classes was not viable for these data, as the classes would have ended up potentially overlapping each other and the introduction of positional uncertainty would have caused confusion and visual clutter in the map.

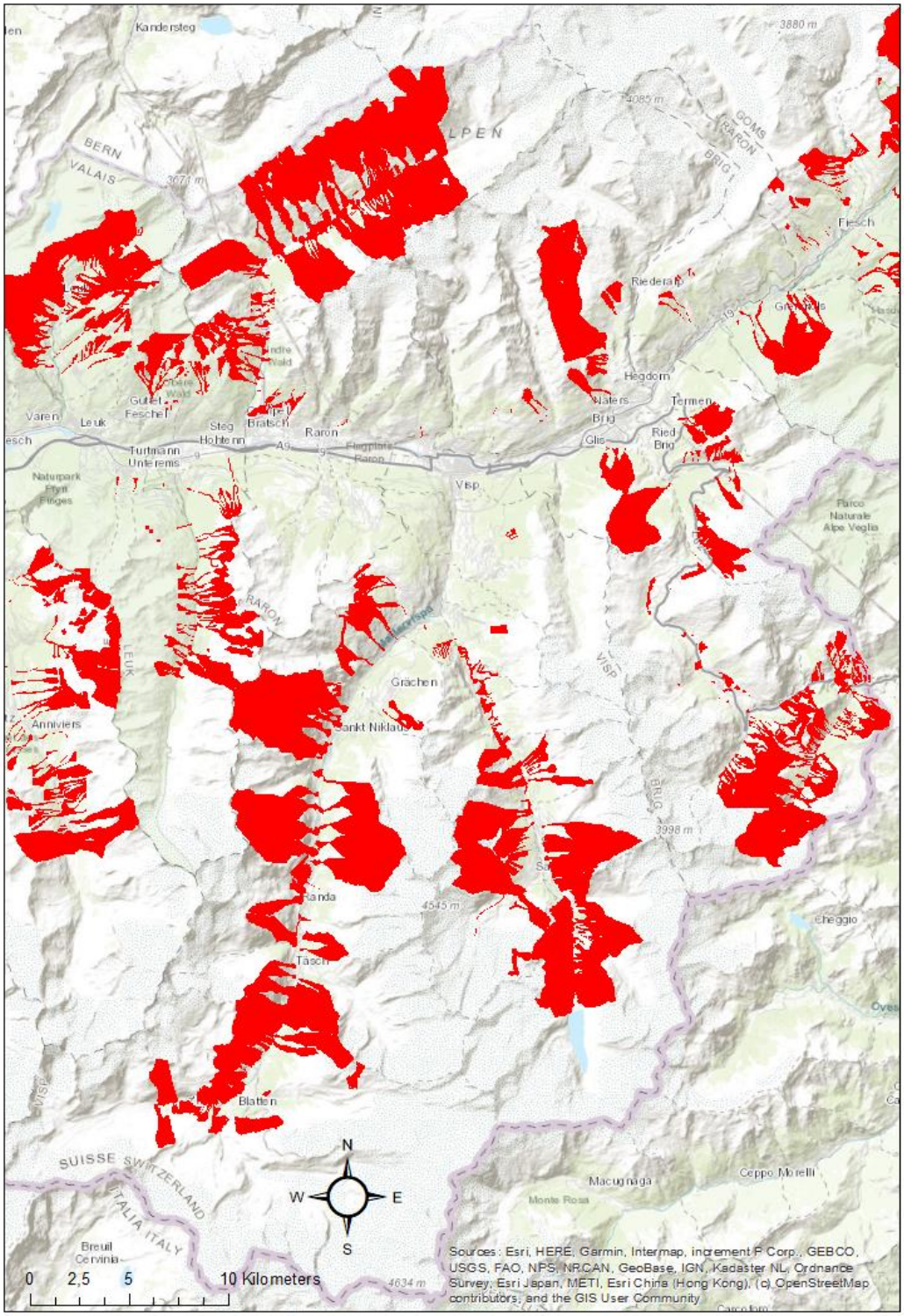
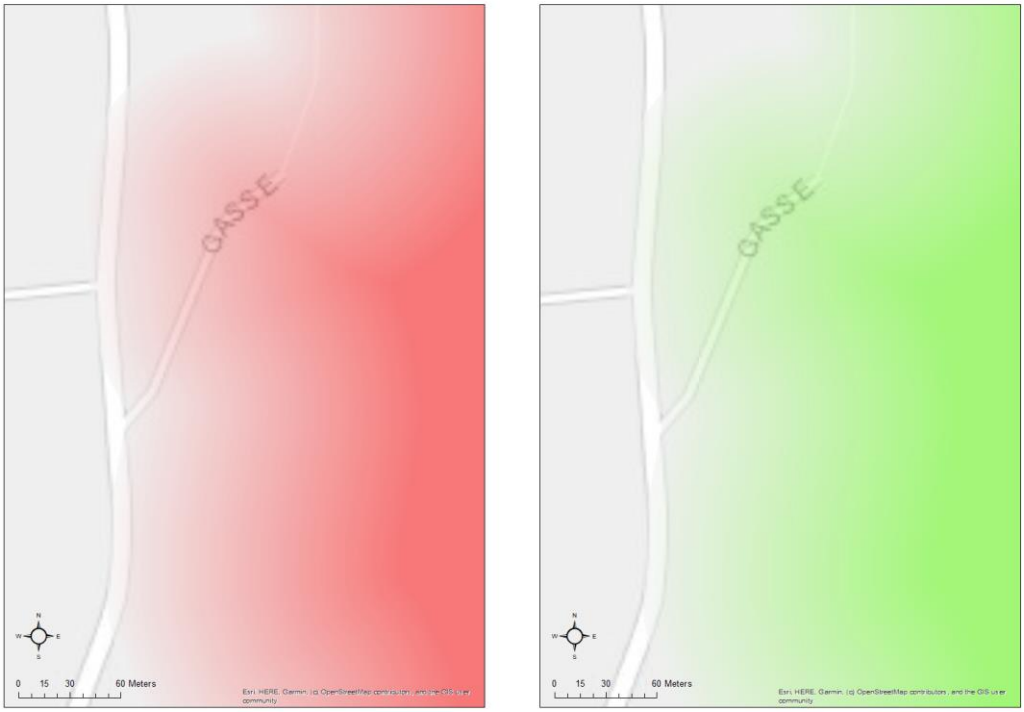


Figure 3.9: A subset of the Naturgefahren-Hinweiskarte of the canton of Valais-Wallis. Red areas are at a high risk for avalanches.

As in the PM10 case study, the addition of two extrinsic variables proved effective to visualize positional uncertainty while maintaining information clarity. The first experiment is a black hatching texture layer superimposed on the 100-m positional uncertainty buffer (Fig. 3.11). Retchless & Brewer (2016) had already tested several types of textured layers to visualize prediction uncertainty in climate change maps, while Johannsen & Fabrikant (2018) used dots and hatches with varying density for uncertainty in precipitation maps. Korporaal & Fabrikant (2019) applied a similar technique to display terrain uncertainty. Unlike the aforementioned gradient, this texture layer covers both sides of the 100-m buffer.

The second experiment is a grey "foggy" layer (Fig. 3.12). MacEachren (1992), who first introduced it among the most significant visual variables to represent uncertainty, described as a layer that "in effect, looks like a fog passing between the analyst and the map" (MacEachren, 1992, p. 14). This layer also covers both sides of the 100-m positional uncertainty buffer, and it is relatively transparent in order not to fully hide the high-risk area visually located behind it and create excessive visual contrasts.

Finally, each one of the three maps also has a version where the high-risk area has a green colour instead of red (Fig. 3.13-3.15). Unlike red, green is not a characteristic colour to represent danger. Additionally, people with color-blindness typically have trouble distinguishing between the two hues (Jenny & Kelso, 2007). Therefore, representing natural hazards with a green colour and confronting these maps with the red ones can provide additional findings on risk perception and heuristic use.



a *b*

Figure 3.10: Gradient borders in the avalanche maps in red and green.

a *b*

Figure 3.11: "Foggy" layer borders in the avalanche maps in red and green.

a *b*

Figure 3.12: "Texture" layer borders in the avalanche maps in red and green.

a *b*

Figure 3.13: Gradient border maps in red and green. The point order is randomized to ensure unbiased ranking.

a *b*

Figure 3.14: "Foggy layer" maps in red and green. The point order is randomized to ensure unbiased ranking.

a *b*

Figure 3.15: "Texture layer" maps in red and green. The point order is randomized to ensure unbiased ranking.

3.3 User test

Background: research methods for user tests

User testing is widely believed to be an essential part of cartographic research; in fact, listening to users' needs and perceptions is crucial for visualization evaluation (Roth et al., 2017). The idea of *user-centered design* has played an increasingly important role in the field, especially since the advent of interactive cartography (Roth et al., 2015). Van Elzakker (2004) argued that, as map communication is an inherently cognitive process, cartographers should always be able to adjust their visualizations according to users' needs. Van Elzakker (2004) also pointed out the difference between *functional map research*, where a sample of potential final users test the maps through specific tasks to provide direct insights on how to improve the product, and *perceptual/cognitive map use research*, where cartographers analyse how users reason upon maps and reach conclusions through visualizations. MacEachren (1994) introduced the idea of a map use cube to visualize the different goals for the use of a map (Fig. 3.16); each goal requires its own evaluation method.

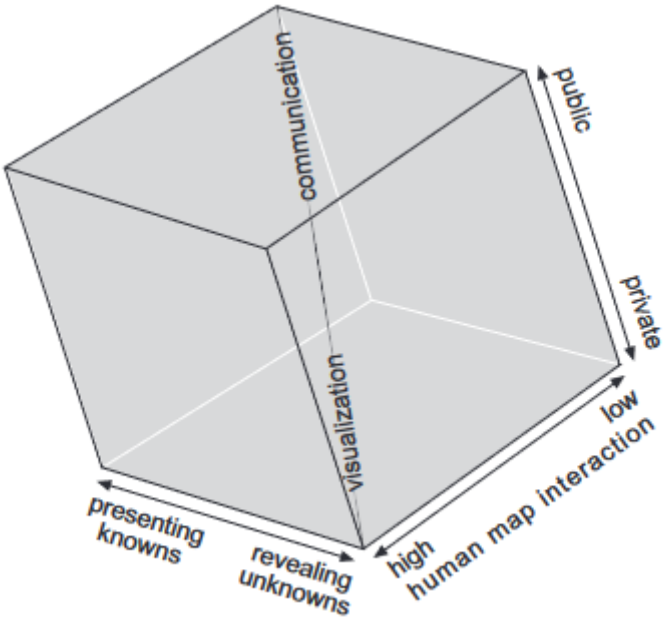


Figure 3.16: The map use cube as proposed by MacEachren. Adapted from Van Elzakker (2004, p. 10).

Both functional and perceptual map research are closely intertwined with the concept of *usability*, described by the ISO 9241 as the effectiveness (ability to perform tasks accurately as intended), efficiency (ability to perform tasks quickly and easily) and satisfaction that users can achieve from a product. *Usability testing* therefore has the goal of improving the usability of a product (Dumas & Redish, 1999); a variety of methodological options is available for this purpose (Paz & Pow-Sang, 2016).

Evaluation through user tests is especially relevant in the context of uncertainty visualization. Aerts et al. (2003) argued that surveys are the only viable solutions to effectively understand which techniques to use in order to visualize different kinds of uncertainty variables for different purposes. Hullman et al. (2019) outlined a methodology for the evaluation of uncertainty visualization techniques that takes into account the different end goals of each study. The authors cite two primary research aims that an uncertainty visualization evaluation can have:

- Behavioural targets, where the study evaluates metrics commonly associated with usability aspects such as effectiveness, efficiency and satisfaction;
- Expected effects, where the research evaluates whether the visualizations elicit a specific response from the user (e.g., presence of biases, or decision accuracy).

Therefore, these studies can have the goal either to evaluate map usability itself or to understand how and why specific maps tend to produce certain emotions and perceptions. Finally, the authors also list several metrics that can be used to actually perform the evaluation, e.g., numerical measures (such as error or variance) or semantic ones (such as risk aversion or affective association).

Van Elzakker (2004), Paz & Pow-Sang (2016) and Dumas & Redish (1999) provided a list of common evaluation tools that apply both to research and usability in general and to the cartography realm, among which:

- Strictly defined *user testing*, where users perform a set of tasks to evaluate how the product conforms to expectations; this can also happen as a *think-aloud session*, where users verbalize their thoughts on the product during the test;
- *Surveys, questionnaires* and *interviews*, where users directly answer to specific key questions on the product or research object; interviews can be structured or unstructured, that is, with pre-determined or spontaneous questions (see also Kumar, 1999);
- *Focus group*, where a subset of representative users discusses the product in a moderated meeting;
- *Heuristic evaluation*, where researchers measure how the product performs in respect to a standard series of usability principles, such as consistency and flexibility of use;
- *Eye tracking*, where researchers analyse users' eye movements during the experiment to detect potential patterns in map use and focus.

Keeping in mind the distinction proposed by Dumas & Redish (1999), the present thesis is a user research rather than an actual usability study, as it does not evaluate the usability of a product but instead looks for the existence of certain cognitive phenomena. From this perspective, it also belongs to the *cognitive and perceptual map use research* as defined by Van Elzakker (2004), as it aims at investigating "why" and "how" users form their knowledge through map visualizations. However, it does share some goals with usability studies, as it also aims at evaluating which visualization techniques for uncertainty perform best in communicating their information effectively.

The tool of choice for the final evaluation of the maps described in the previous subchapters was an anonymous online questionnaire, which is one of several web-based methods that have been becoming increasingly popular for evaluation studies in cartography in recent years (Kinkeldey et al., 2014). Questionnaires present several advantages and limitations: Martin (2007) argues that surveys and especially online surveys are highly cost-effective and easy to administer to a broad audience in a short time, but it may be difficult to control for

sampling biases. In this context, an online questionnaire presented additional advantages in light of COVID-19-related research constraints, such as laboratory shutdowns and the inability to meet test participants in person.

Questionnaire setup and administration

The concept and structure for the questionnaire follow methodological advice and guidelines proposed by Bryman (2001), Martin (2007) and Luz et al. (2017).

Participants and apparatus

Sixty-one users (of which thirty-three females), recruited through academic and personal networks, took part in the test. Users' age, gender, education title and level of expertise with maps and natural hazard datasets were all controlled for, as they could potentially highlight different patterns in answers (e.g., older respondents being more conservative in their ratings). Users showed relatively high age diversity, with eleven being younger than 25, twenty-five between 25 and 30, fifteen between 31 and 35 and ten older than 35. 85% (n=52) of users had a university degree, with four others having a tertiary non-university degree (e.g. Fachhochschule) and five having a secondary school title or less. Users reported having a relatively average level of expertise with maps, with a self-assigned mean rating of 3.71 in a scale from 1 (very low level of expertise) to 5 (very high). Their self-assigned average level of expertise with the natural hazard dataset was only 2.41 on the same scale.

The platform of choice to build and host the questionnaire was SoSciSurvey, an online tool for social research that the Technical University of Munich offers free of charge to its students and employees. The platform itself as well as the questionnaire's characteristics allowed users to keep full anonymity and complete the questionnaire at any time; this helped reduce the likelihood of any self-selection and non-response bias.

Variables, tasks and analysis of results

As previously stated, the questionnaire was chiefly aimed at evaluating users' perceptions and responses to maps depicting uncertainty and risk areas with several different "borderization" choices. Therefore, the primary *independent*

variable – the variable whose effects are the subject of the investigation – was the "borderization" itself, while the *dependent variable* – the variable that serves to measure the effects of the independent variable – was the users' perception of the maps. As previously discussed, maps depicting natural hazards are often used in cartography to evaluate heuristic use and cognitive perception; therefore, rating natural risk levels seemed an appropriate task for the survey.

In more detail, the questionnaire consists of 25 core questions, further divided into several sub-questions. Each user viewed a total of seven maps, four from the first case study (the PM10 dataset), and three from the second case study (the avalanche dataset). The questions were randomized to assign users to two different groups, each one visualizing two pairs of "borderizations" from the PM10 case study (with and without uncertainty information) and three visualizations from the avalanche dataset – one in red without extrinsic uncertainty and two in green with extrinsic uncertainty and vice versa. As such, each visualization could be included with the same frequency in the final set of working questionnaires without making the questionnaire overly long. Additionally, this solution was useful to prevent users from seeing too many maps, which could have caused a "task order effect", i.e., the observed improvement and/or increased awareness in survey results that sometimes happens after a few repeated tasks (Tullis & Albert, 2008). To further prevent the occurrence of such an effect, users did not have any information about the real subject of the study and were only told that the survey dealt with the perception of natural risk hazards.

Each map included a series of potential housing locations across several categories of natural hazard risk as explained in subchapters 3.2.2 and 3.3.2. Users were asked to pretend to be looking for new homes in locations as safe from pollution hazards or avalanche risk as possible. They would subsequently rate the perceived levels of desirability or safety, respectively, using a standard Likert-type scale with ten options from "not desirable at all" (or "completely unsafe") to "highly desirable" (or "completely safe"). On the same page, they were

also asked, in an open-ended question, to describe the main reasons behind their ratings in two or three brief sentences. (Fig. 3.17)

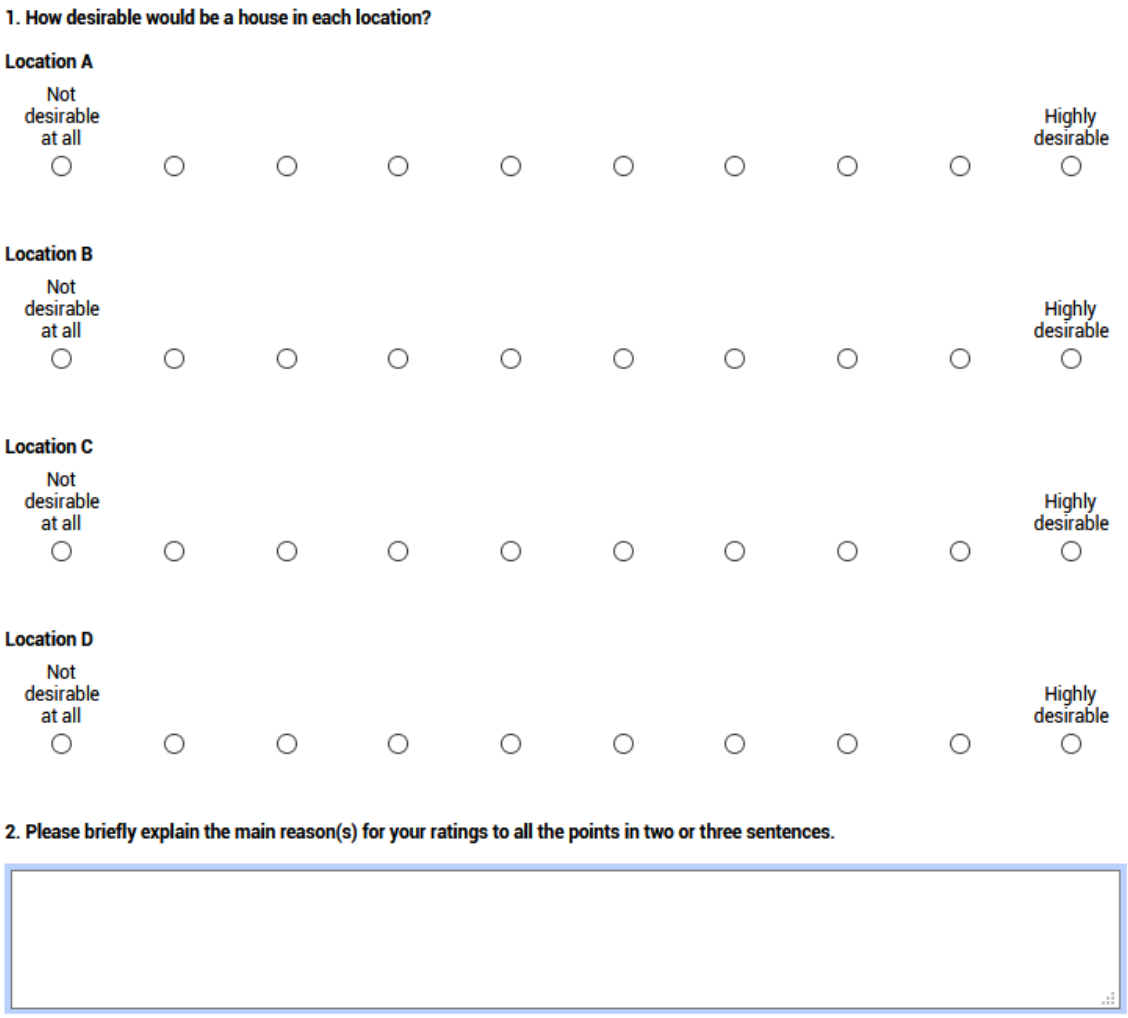
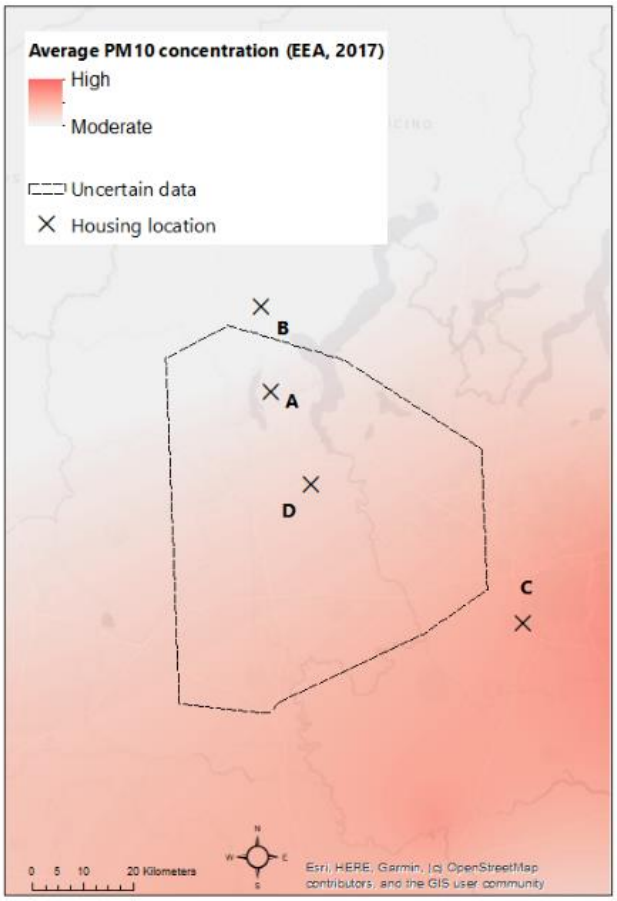


Figure 3.17: Standard point rating and open-ended questions in the questionnaire.

Finally, users provided their levels of confidence in such ratings using the aforementioned Likert-type scale. Their ratings were subsequently counted as numbers from 1 to 10. Housing locations had a randomized letter order, so as to avoid triggering the "adjustment to an anchor" heuristic and prevent users from simply applying the same ratings to each map regardless of the specific visualization. At the same time, all the questions had the same structure and included the same rating scale, so as to provide high comparability. (Fig. 3.18)



1. How desirable would be a house in each location?

Location A

Not desirable at all ○ ○ ○ ○ ○ ○ ○ ○ ○ ○ Highly desirable

Figure 3.18: Example of questionnaire page including map with randomized locations.

After these questions, a new page would include four dichotomic questions where users have to select one map from a pair according to how intuitive, effective and/or satisfactory they felt it to be. The goal behind the inclusion of this page was to gather insights about the usability of each visualization technique, i.e. its suitability for conveying the information displayed.

On the last page, users simply had to rank several colour shades according to their intuitive association with an idea of "risk", from lowest to highest. This was meant to control for the potential presence of colour-blind users, whose altered

colour perception could provide additional unexpected findings for the scope of the survey.

The nature of the study is, therefore, both quantitative and qualitative. Ratings and confidence levels are treated as numbers from 1 to 10 and their distribution is the subject of descriptive statistical analysis (e.g., mean and standard deviation) and statistical significance testing. As the ratings do not present a normal distribution, therefore making T-test or Z-scores impractical, the non-parametric alternative Wilcoxon-Mann-Whitney is an effective tool to test for significance between different point rating averages.

On the other hand, open-ended answers are the subject of qualitative analysis, in order to highlight potentially relevant statements that would support conclusions from the significance testing. The goal was to investigate whether (and, if so, how) different "borderizations" would significantly alter users' ratings and confidence levels and whether relevant heuristics such as containment or distance could be the driving forces behind these differences. However, this qualitative search proved somewhat difficult to perform as keyword use was often ambiguous, especially concerning containment-related biases. To reduce potential over- or under-counting issues and come up with as an objective analysis as possible, statements were considered relevant only when keywords were part of a larger and more meaningful sentence structure. For instance, the use of "in" from the statement "I feel really uncomfortable in an air pollution area" appeared too generic to be referred to a containment bias with any degree of confidence. On the other hand, the statement "C and D are located within the high concentration zone" seemed to suggest that its author did indeed use a containment heuristic to rate the two points specifically. Subsequently, the statements were counted and cross-referenced with point ratings to try and evaluate whether they would further support previous conclusions or not.

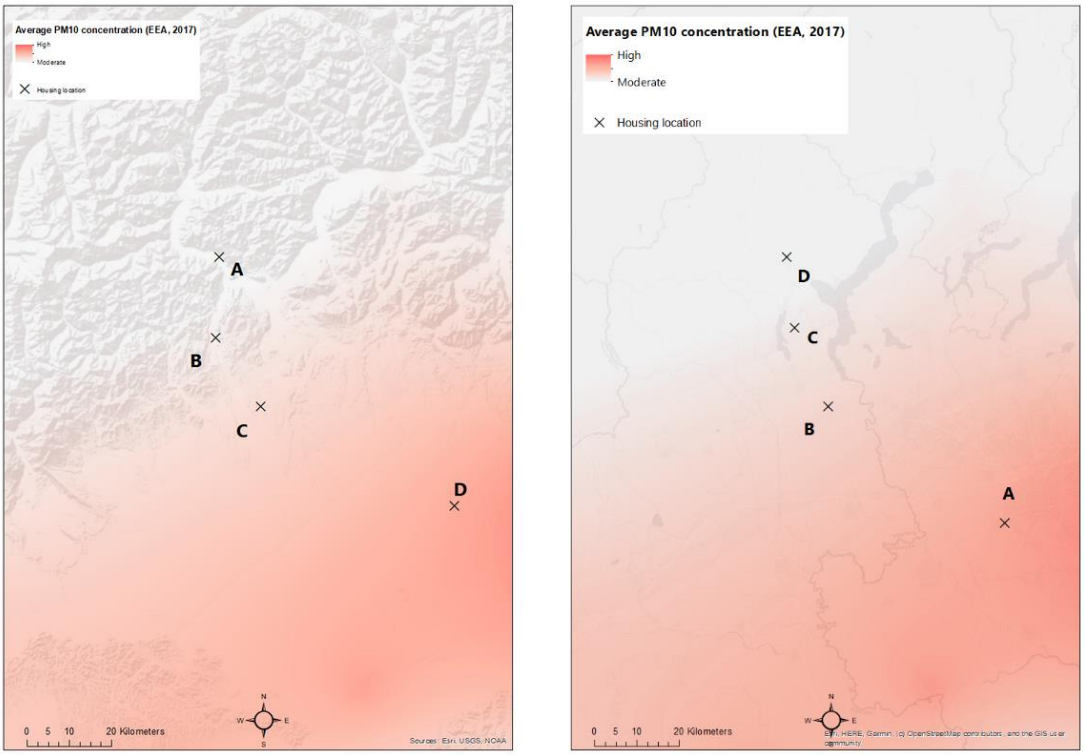
Pilot study

After the questionnaire setup, the following step was to perform an exploratory pre-test on a small sample of relevant users, in order to gather real feedback about the test's overall functionality and possible flaws to be improved. This is

an established practice in cartographic research (see e.g., Leitner and Buttenfield, 2000). Eight individuals with high levels of cartographic expertise participated in the pre-test, which took the form of a semi-structured one-on-one interview on the online video service Zoom. These eight individuals first performed the survey while commenting freely and without any guidance on the visualizations and questions they were seeing; having finished, they then answered several more direct questions about the maps and the survey structure. They provided useful feedback to improve and finalize the questionnaire:

- Map legend was too small and barely readable, especially on smartphone screens;
- The hillshade basemap, initially chosen as background in the PM10 visualizations, was confusing and added unnecessary biases; therefore, the final maps have a more neutral light grey basemap with minimal labels (Fig. 3.19);
- The questionnaire lacked a clear definition of "uncertain data", as some users could not understand whether the label referred only to the dashed line or to the area inside it and in which terms; questions were subsequently rephrased to explain how points within the uncertain area may belong to the closest higher or lower risk category;
- It was better to randomize the order of housing location labels, in order to avoid triggering the "adjusting to an anchor" heuristic as previously explained.

Users' answers also suggested that the expected biases were indeed present. Namely, the containment and distance biases seemed to be stronger in the "hard border" maps and in the maps with extrinsic uncertainty. "Fuzzy" borders increased the amount of information but also confusion and perceived uncertainty; in other words, they did not necessarily cause a change in ratings, but they did seem to decrease confidence levels. "Fuzzy" borders also seemed to provide a better idea of the geographical distribution of natural hazards, especially in the avalanche risk maps. Chapter 4 analyses in detail the actual results of the finalized questionnaire.



a *b*

Figure 3.19: Example of a PM10 before (a) and after pre-test (b). The basemap was changed to a more neutral background.

3.4 Summary of methods

This chapter has presented the methodology adopted to build the whole study and fulfil its objectives.

Subchapter 3.1 has introduced the main challenges related to detecting the use of heuristics in cartography, and subsequently the strategies used in the present thesis to try and tackle these pitfalls by producing objective analysis criteria.

Subchapter 3.2 has dealt with the design of the maps included in the study, firstly by explaining their logic and conceptualization and secondly by showing the techniques adopted to come up with the final cartographic products.

Finally, Subchapter 3.3 explained the different stages of building the online user survey, from choosing the platform and the participants to shaping the questions and performing the exploratory pilot-test.

4. Results and discussion

4.1 Structure of results

This chapter will show the results of the user test, in light of previous literature findings on heuristics and uncertainty visualization related to the issue of borderization.

Subchapter 4.2 will list in detail the point rating results from the main section of the survey in both case studies, and will draw tentative conclusions about their significance for the scope of the study. Findings on confidence levels will be the subject of Subchapter 4.3.

Subchapters 4.4 and 4.5 will present further results from the survey sections on one-to-one comparisons and colour shades ranking. These results will also be interpreted as support for the conclusions previously made.

Subchapter 4.6 will then show the most relevant statements from the open-ended questions. These statements will be listed in reference to the findings from all the previous subchapters, in order to further provide solidity and strength to their conclusion.

Finally, Subchapter 4.7 will summarize all the findings theme by theme, cross-linking them to the original study objectives in order to provide a final answer to the research questions.

Table 4.1 echoes Table 3.1 in Subchapter 3.1.1 by showing initial specific assumptions on the expected relation between heuristics and survey results in terms of users' behaviours, perceptions and response patterns.

Heuristic	Assumption(s)
Distance	• Significant difference in average ratings between two points in the same thematical category (e.g., colour class) but differing in their distance from a thematical border • Lower differences among point ratings with fuzzy borders than with hard borders • Significant rating alteration for extreme points (reduction for "safe" ones, increase for "unsafe" ones) after the introduction of extrinsic uncertainty

Containment	• Higher ratings for safe points and lower ratings for unsafe points with hard borders • Lower difference between safe and unsafe points with fuzzy borders than with hard borders • Lower difference between points within uncertain area after its addition to the map (PM10 maps)
Availability	• Reported difference in perception between PM10 maps and avalanche maps due to lived experience
Adjustment to anchor	• Repeated rating patterns across successive maps • Repeated rating patterns across successive maps from one user despite significant changes in ratings from other users
Representativeness	• Lower ratings for unsafe points when risk areas are coloured in red • Preference for maps depicting PM10 risk with a fuzzy border

Table 4.1: Specific initial assumptions on the relation between heuristics and users' behaviours and perceptions.

4.2 Effects of borderizations and extrinsic uncertainty on heuristics

PM10 maps: point ratings without extrinsic uncertainty

In the PM10 maps, users in both groups rated points from A to D with decreasing levels of desirability. This effect appeared across all maps, regardless of the borderizations used. In the "Hard Border" maps - both single and layered, with and without extrinsic uncertainty -, for example, point A received an average desirability rating of 7.65 out of 10, whereas point B had an average rating of 5.28. Points C and D, both lying entirely outside the high PM10 concentration area, had average ratings of 2.56 and 1.57 respectively. The differences between A, B, C and D in the "Hard Border" maps were statistically significant (p-value < 0.01). The same differences are even larger in the two "Single Hard Border" maps, with point A and B having average ratings of 8.28 and 6.00 in the map without extrinsic uncertainty. These findings seem to show evidence of a distance bias. In fact, the "Hard Border" maps do not give any information about the distribution of PM10 levels both within and outside the high concentration areas, therefore users have no factual basis for assigning lower ratings to B compared to A or D compared to C. On the contrary, distance from the border alone seems to be a driving force in desirability evaluation. Additionally, as the "Hard Border" maps were the first to appear in the survey, users were not biased by other visualizations.

In the visualizations without extrinsic uncertainty, ratings for point A significantly decreased from "Single Hard Border" to all the other borderizations (p-

values < 0.05). Ratings for point B decreased from “Single Hard Border” to “Limited Fuzzy Border” (p-value = 0.02) and stayed the same in the other two borderizations. Conversely, ratings for point C *increased* from “Single Hard Border” to “Limited Fuzzy Border”, while ratings for point D increased from “Single Hard Border” to “Total Fuzzy Border”. Overall, there seemed to be a containment effect at play: users felt safer in “unsafe” points and less safe in “safe” points when the high PM10 concentration area was visualized with a “fuzzy” border instead of a single hard border, as well as with a layered border although the effect was less pronounced in that case (see Fig. 4.1).

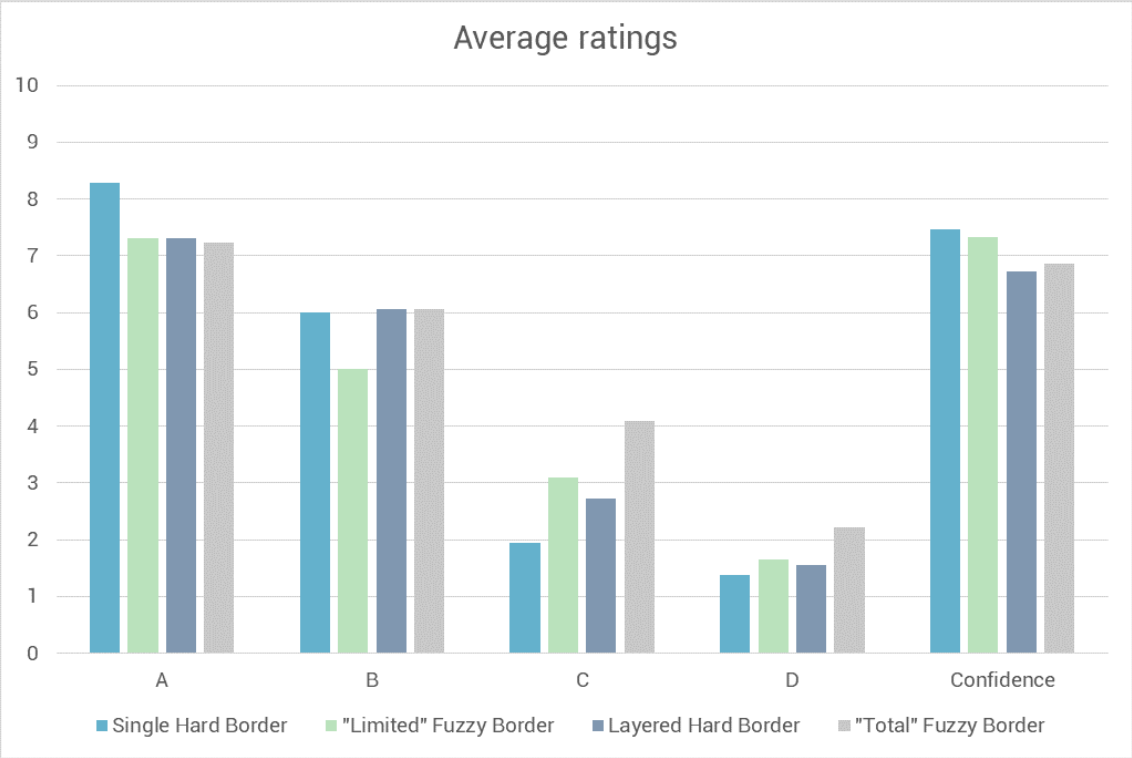


Figure 4.1: Average ratings of the points and associated confidence in the four borderizations without extrinsic uncertainty.

PM10 maps: point ratings without vs. with extrinsic uncertainty

The visualization of extrinsic uncertainty affected desirability ratings across all borderizations, although its effects were not homogeneous.

Ratings for point A significantly decreased (p value = 0,019) in the “Hard Border” maps after the introduction of uncertainty, even though A lies outside the uncertain area. This further suggests the presence of a distance bias, as A was closer to the uncertain area than to the border of the high concentration area in the map without uncertainty (Fig. 4.2).

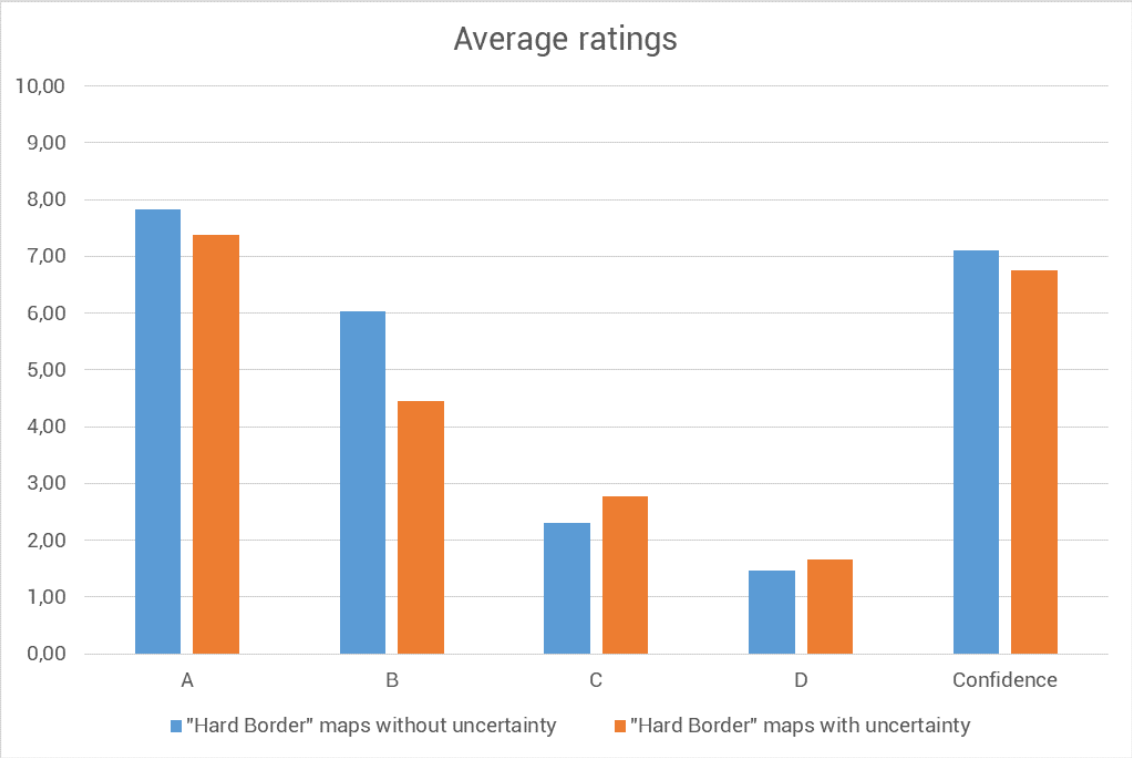


Figure 4.2: Ratings for "Hard Border" maps with and without uncertainty.

Ratings for B decreased (p value < 0,01) in all four borderizations, while ratings for C increased (p value = 0,02) in “Layered Hard Border” after the introduction of uncertainty. Ratings for D did not show significant variations. Overall, the effect of uncertainty was stronger in the “Hard Border” maps than in the “Fuzzy Border” maps. As the ratings of A and B in the “Single Hard Border” map heavily affect the average total ratings in the “Hard Border” maps, it seems that the introduction of uncertainty lessened the effect of the initial containment bias for A and B, while generating further biases. In other words, the introduction of uncertainty shifted the focus of the containment bias from the high concentration area to the uncertain area. In the “Fuzzy Border” maps, this effect was less prevalent as users focused more on the distribution of colours instead (Fig.4.3).

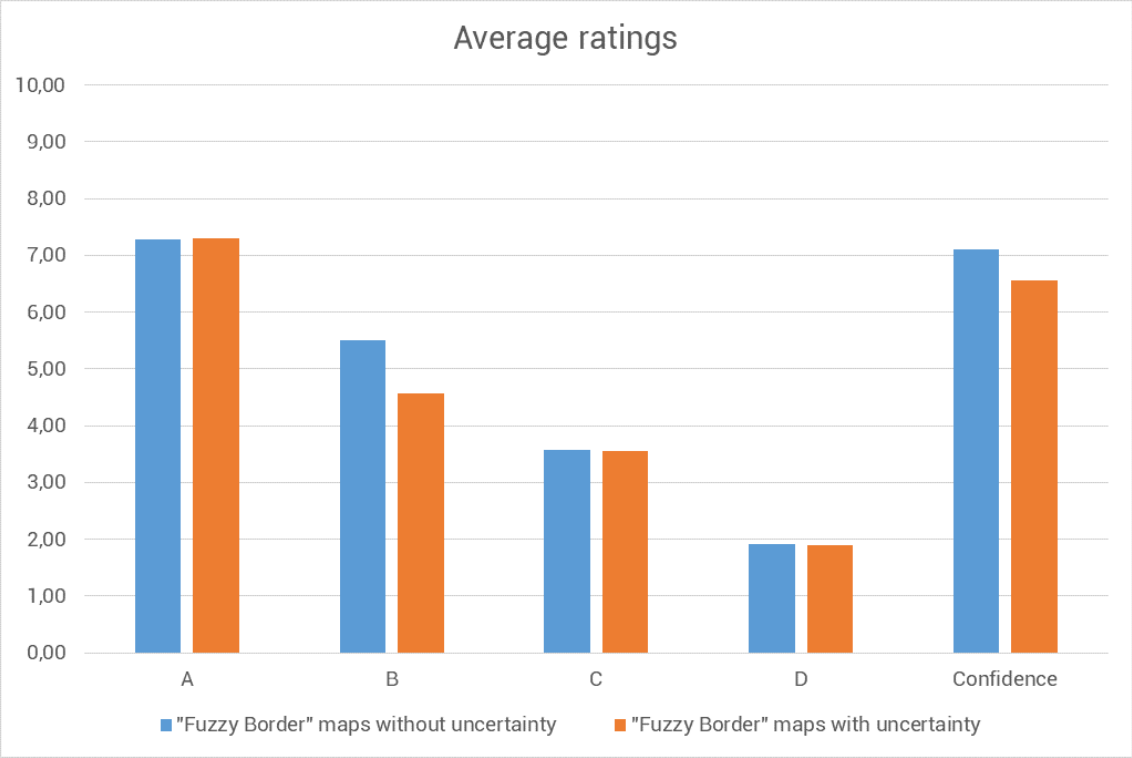


Figure 4.3: Ratings for "Fuzzy Border" maps with and without uncertainty.

Interestingly, while ratings for A decreased in all the borderizations compared to "Hard Border" when uncertainty was not present, the same did not happen in the visualizations with uncertainty. In fact, ratings for A did not show significant variations across the four borderizations with uncertainty. The rating in "Limited Fuzzy Border" (7.63) was even slightly higher than in "Single Hard Border" (7.59), although the difference was not statistically significant.

This suggests that the introduction of uncertainty heavily reduced the effects of containment bias, possibly, as previously mentioned, by increasing the overall feeling of risk in the 'safe' areas. Furthermore, the standard deviation of the ratings in "Hard Border with uncertainty" is decidedly higher (2.98) than in "Hard Border without uncertainty" (2.16). In other words, the introduction of uncertainty not only increased risk perception for A but also caused large intra-sample variations in ratings. This also suggested that, in the map with uncertainty, users' responses were less driven by the same containment bias which had made decisions easier and more clear-cut in the map without uncertainty.

Ratings for B, C and D further confirmed these findings. Ratings for B showed a significant decrease (p-value <0.044) from "Hard Border" (6.03) to "Fuzzy Border" (5.51), but only without extrinsic uncertainty. At the same time, as previously mentioned, ratings for B decreased in *all* borderizations after the introduction of uncertainty. This again shows how extrinsic uncertainty tended to negatively affect the rating of "safe" points and had a larger impact overall than the borderization itself and the choice of colours. Unlike in A, however, standard deviation did not show any detectable patterns, with a few small changes being likely due to the different samples of respondents (Fig. 4.4).

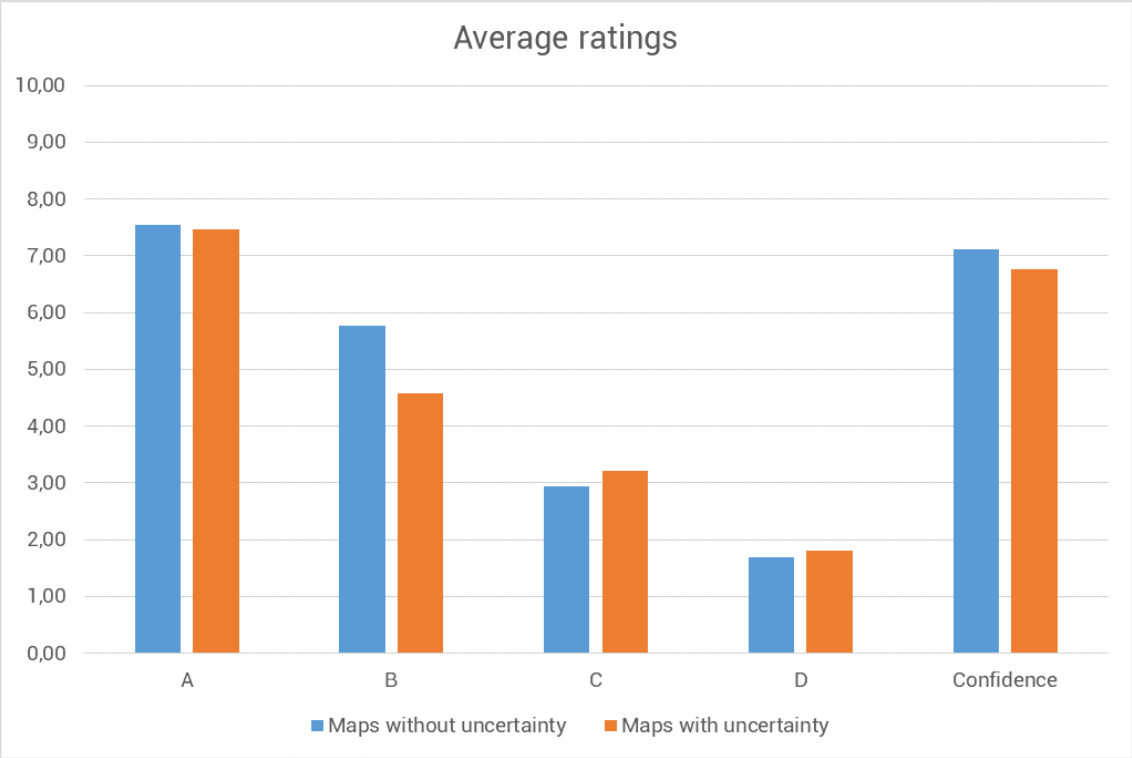


Figure 4.4: Ratings of points in all maps, with and without uncertainty.

On the other hand, ratings for C increased from "Hard Border" to "Fuzzy Border" even when uncertainty was present, and in the case of "Layered Border" they also increased in the visualization with uncertainty compared to the one without uncertainty visualized. Ratings for D did not show increases after the introduction of uncertainty. However, they did significantly increase in "Total Fuzzy Border" compared to "Layered Border" both with and without uncertainty present (p-value = 0.026 and = 0.020 respectively). In short, uncertainty did not cause

any significant decrease in ratings for C and D, unlike A and B, and in some cases even produced an increase. Furthermore, ratings for C increased from “Hard Border” to “Fuzzy Border” even when uncertainty was present, while ratings for A decreased from “Hard Border” to “Fuzzy Border” only without uncertainty (Fig. 4.5-4.6).

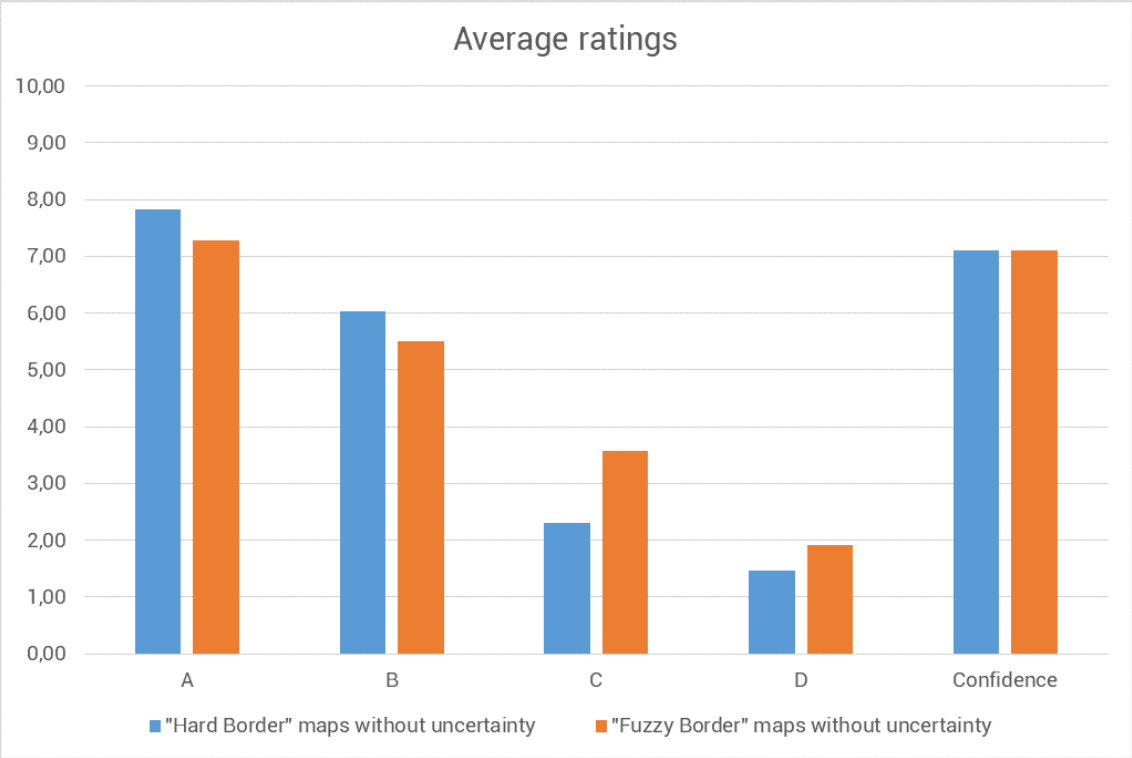


Figure 4.5: Ratings for points in "Hard Border" and "Fuzzy Border" maps without uncertainty.

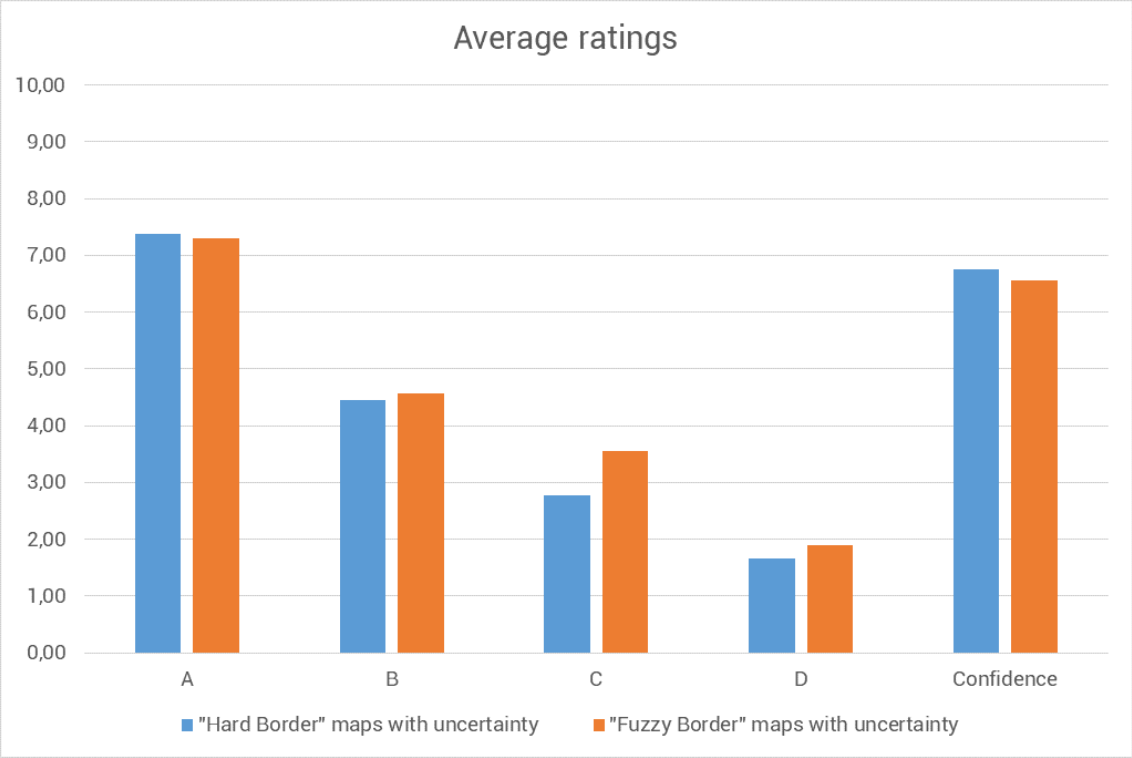


Figure 4.6: Ratings for points in "Hard Border" and "Fuzzy Border" maps with uncertainty.

This effect suggests the presence of a *loss aversion bias*. As previously mentioned, the introduction of uncertainty seemingly caused users to feel less safe in the same areas that, due to the containment bias, they had previously deemed safer. At the same time, this pattern does not reflect a tendency towards risk aversion, as ratings for C increase even though C is located in the uncertain area. Therefore, users appeared to overweigh the potential gain produced by uncertainty around C over the risk that the uncertainty itself created, while the same did not happen in B. This is consistent with a loss aversion pattern as mentioned in Tversky & Kahneman (1979).

Finally, it is also notable that, as for point B, variations in standard deviation for both C and D were less significant compared to A. In the case of C, only the standard deviation for “Single Hard Border” without uncertainty was decidedly lower than the other ones, whereas standard deviation for D showed erratic patterns with no clear tendencies detectable. In fact, uncertainty did seem to increase standard deviation in ratings to D in “Single Hard Border”; at the same time, both the visualizations of “Total Fuzzy Border” showed higher standard

deviations in ratings compared to the other visualizations. Such an effect also produced a modest increase in the ratings themselves, which may be caused by a small sample size effect with few uncharacteristic ratings skewing the entire result. Fig. 4.7 shows the average ratings for all the points in each borderization, with and without uncertainty.

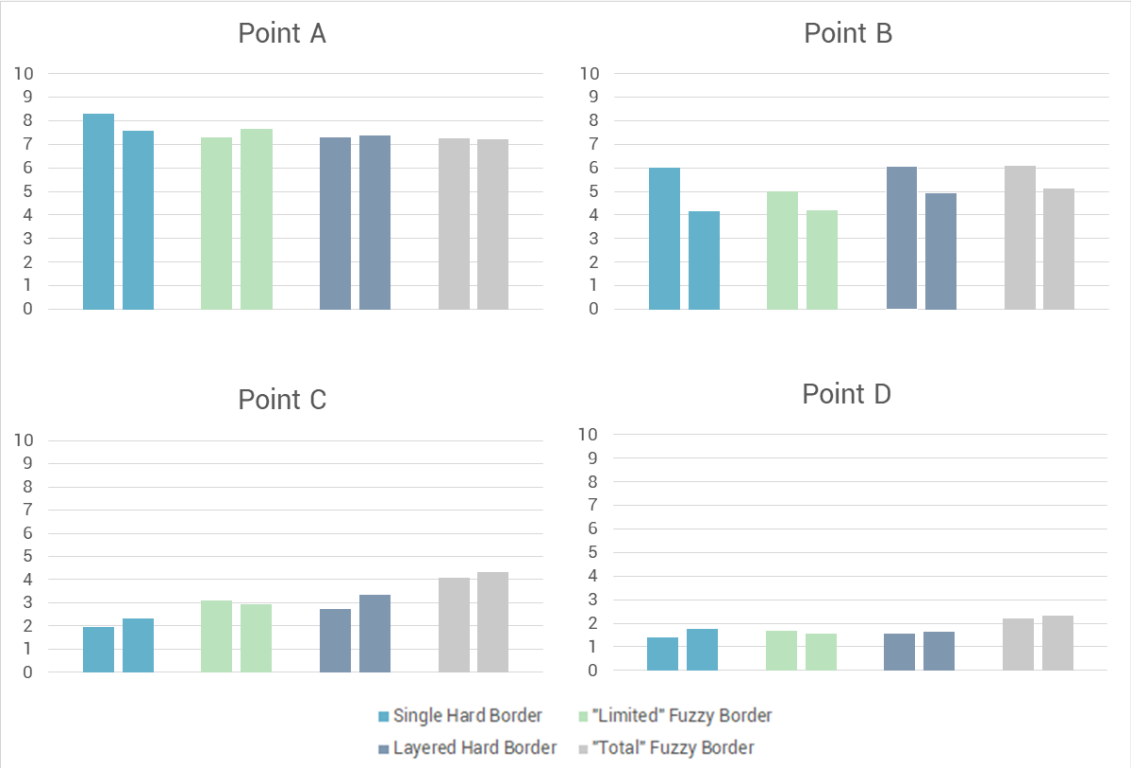


Figure 4.7: Ratings for all the points in each of the four borderizations. Columns on the right refer to borderizations with extrinsic uncertainty.

Avalanche maps

In stark contrast to the PM10 maps, point ratings in the avalanche maps showed far fewer significant patterns. Most differences happened between the two samples of respondents rather than between different maps presented to the same sample.

Distance bias was undoubtedly present in all the maps, as in the previous study case. Even if both point A and point B were located well outside of the risk area,

point A had an average rating of 8.34 across the six different maps, compared to 7.09 for B. Similarly, point D and E had different average ratings across the six maps (1.89 and 1.38 respectively) despite both of them being located inside the risk area. Once again, the maps did not provide any additional information on hazard levels inside and outside the high-risk area; therefore, such changes in ratings could only be caused by an intuitive association between risk levels and distance from the border of the high-risk area. Even more tellingly, "Fuzzy" maps displayed the same difference in ratings between D and E as the maps with extrinsic uncertainty, even if the background colour in the former was exactly the same for both D and E. This further shows that users intuitively felt safer in D because D, while still lying inside the high-risk area, was located closer to its border.

Ratings for point A did not show any significant intra-group differences. Ratings for point B were significantly lower (p-value = 0.04) in "Green with Texture Layer" (6.25) compared to "Green with Foggy Layer" (6.88). Nevertheless, as this effect did not occur in any other map, it is hard to highlight a single motive behind the result. The standard deviation of ratings for B in "Green with Foggy Layer" was slightly higher (2.93) than in "Green with Texture Layer" (2.81), although lower than in "Fuzzy Green" (2.98). Once again, this pattern does not seem to suggest any particular heuristic being at play.

Notably, Group 2 assigned higher ratings overall to A and B compared to Group 1 and also displayed a lower standard deviation. Such differences were not present in any of the maps in the first study case.

While ratings for C did not show any significant differences across maps, D was the point with the largest changes in ratings. In fact, D in "Green with Texture" had a significantly higher rating (2.34, p-value = 0.02) than both "Green with Fog" (1.72) and "Fuzzy Green" (1.66). D in "Red with Layer" also had a markedly higher rating (2.27, p-value < 0.01) than in "Fuzzy Red" (1.34), although the difference was not significant between "Red with Layer" and "Red with Fog". Interestingly, point E also showed a significantly higher rating in "Green with Layer" (1.66, p-value = 0.048) compared to "Green with Fog" (1.25). As "Green with Layer" and

“Green with Fog” were rated by the same sample, the difference in ratings cannot be attributed to a difference in groups. Therefore, when the risk area is visualized in green, the use of a textured layer to extrinsically visualize uncertainty seemingly decreases risk perception in unsafe areas. Some effect can also be seen in the map coloured in red, although only for point D which is located on the texture. This may suggest that, in the case of point E, the strong effect of the red colour in the background overwhelms a possible counter-effect from the texture layer nearby. Additionally, while ratings for D and E both increased in “Green with Layer” compared to the other borderizations, ratings for B decreased; this could be the result of a modest containment bias, where the texture layer has the same effect as the extrinsic uncertainty seen in the PM10 maps.

At the same time, however, ratings for E in “Green with Layer” show a higher standard deviation (2.09) than in the other visualizations. This suggests that the modest effect just described could also simply be due to a few uncharacteristic results that skewed the ratings and caused falsely significant differences.

Differences in gender, age, and expertise with maps

Interestingly, female users consistently assigned lower average ratings to all points across all visualizations compared to males (Fig. 4.8). This result indicates that female users might be more conservative in their ratings compared to males. However, differences between maps were the same between males and females; this suggests that, concerning heuristic use, females did apply the same reasoning patterns as males.

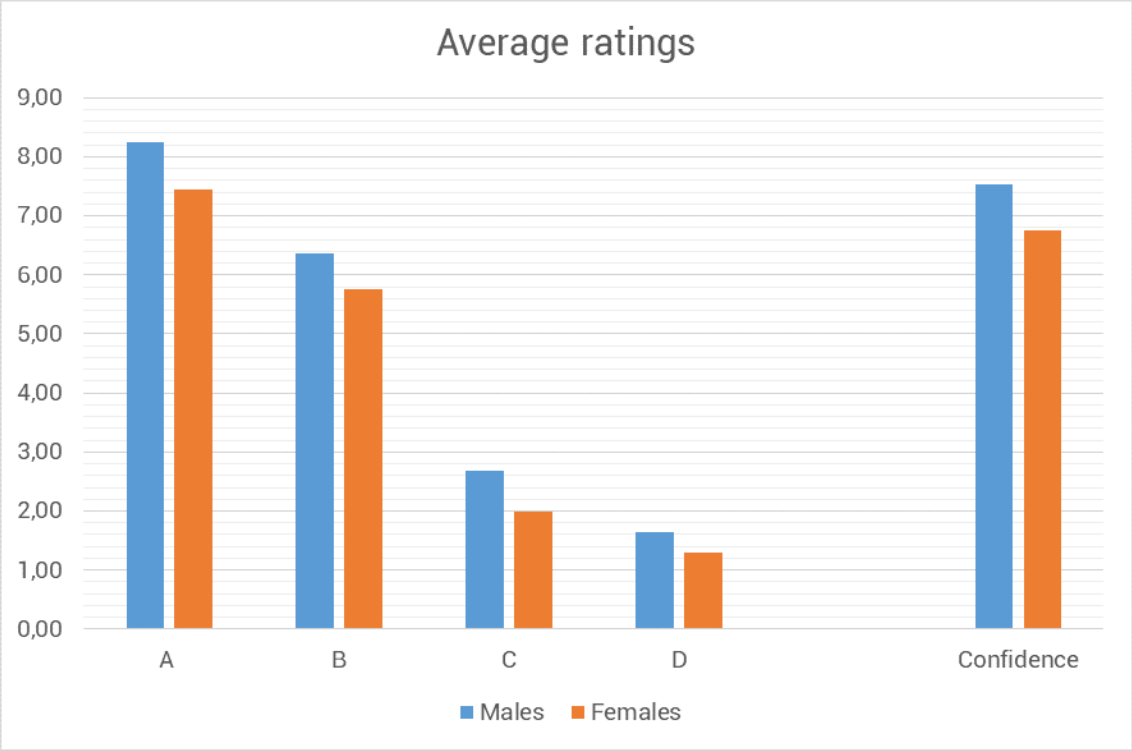


Figure 4.8: Average point ratings in "Hard Border" maps by gender.

Age did not seem to affect rating distribution significantly. Younger users assigned lower ratings in some maps and higher ratings in others seemingly without any readable pattern, often not even within all the points from one single map. This might be due to the skewed age distribution, where the ages between 25 and 35 are overrepresented: only one third of total users were younger than 25 or older than 35, which made any statistical breakdown more uncertain.

Users with a high self-rated level of expertise with maps (above 3 in a Likert-type scale from 1 to 5) tended to assign higher ratings to “safe” points and equal or lower ratings to “unsafe” points than users with a low self-rated level of expertise (3 or below in the aforementioned Likert-type scale). This again might suggest that users unfamiliar with maps felt less confident in their answers and tried to avoid extreme ratings; this is also reflected by the confidence results as shown in Subchapter 4.2. However, it must be noted that most users in this group had a self-rated “medium” level of expertise (3) and few had a very low level.

4.3 Effects of borderizations and extrinsic uncertainty on confidence

The introduction of extrinsic uncertainty in the PM10 maps seemed to cause a decrease in confidence across most borderizations. Indeed, the average confidence for the total of the maps without uncertainty (7.11) was significantly higher than for those with uncertainty (6.76, p value < 0.01). The decrease in confidence following the introduction of uncertainty was also significant (p value = 0.03) for both the "Hard Border" and the "Fuzzy Border" maps. However, in the detailed breakdown by borderization, the decrease was only visible in the "Single Hard Border" and in the "Total Fuzzy Border" pairs (p value = 0.044), whereas both "Layered Hard Border" and "Limited Fuzzy Border" showed slight, non-significant decreases. Standard deviation remained quite homogeneous across all borderizations.

Interestingly, among the visualizations with uncertainty, "Single Hard Border" and "Total Fuzzy Border" were also the ones with the highest and lowest average confidence respectively (Fig. 4.9). There might be two different factors at play to explain the decrease in the two borderizations. As stated above, respondents seemingly made a heavy use of the containment and distance heuristics in "Single Hard Border" to assign their ratings. The introduction of the uncertainty layer may have disrupted their perceptions and rendered these simple heuristics unviable, thus making rating choices much less straightforward – although still relatively easy on a global scale, as Fig. 4.8 shows.

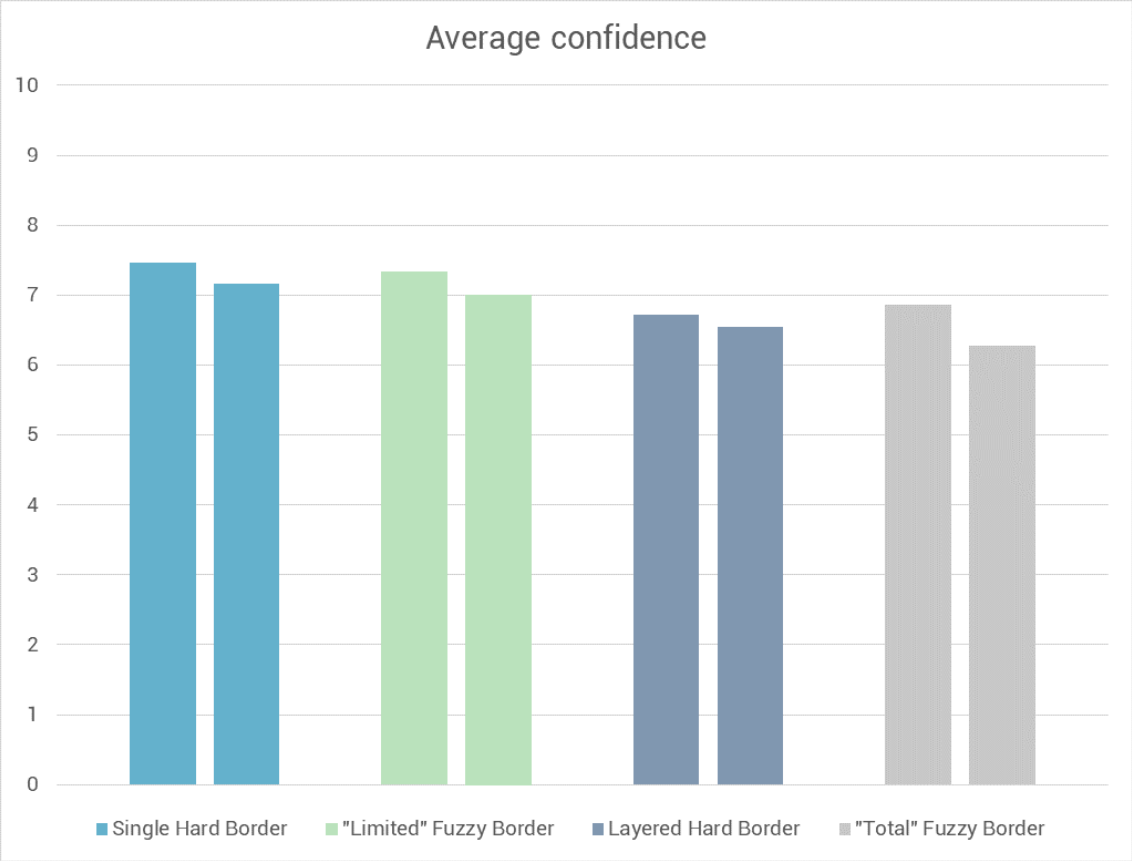


Figure 4.9: Average confidence levels for all the eight borderizations. The two columns show confidence levels in the maps without (left) and with extrinsic uncertainty (right).

As for the "Total Fuzzy Border" map, the slight and subtle gradient encompassing the whole map space may have confused the users from the beginning; the addition of uncertainty could then have made it even more difficult to tell the different locations apart in terms of potential risk. In fact, "Total Fuzzy Border" was the only borderization where viewers did not have any clear boundary to use in order to assign the points to different risk categories; at the same time, the introduction of uncertainty made it hard to gauge potential risk through changes in background colour as well.

A further element to support this observation is that confidence in "Total Fuzzy Border with uncertainty" has the lowest standard deviation among all the visualizations. In other words, the introduction of uncertainty in "Total Fuzzy Border" did not only cause a decrease in confidence, but this decrease was also homogeneous among users instead of being caused by a few outliers.

In the avalanche maps, confidence only showed a significant decrease from "Fuzzy Green" (7.45) to "Red with Fog" (6.97, p-value = 0.032). This effect was only associated with a modest increase in standard deviation, which, however, remained lower than in "Red with Texture". Notably, point ratings showed no significant changes between "Fuzzy Green" and "Red with Fog": as the latter appeared directly after the former in the questionnaire, such a pattern may imply a small availability effect among users, who simply applied similar ratings as before but simultaneously felt less confident about them. No further significant changes in confidence were observed in the other visualizations.

Among personal characteristics, only gender and level of expertise with maps seemed to affect confidence. Females and users with average or low self-rated expertise with maps reported lower confidence across all maps. These patterns resemble those explained in Subchapter 4.2.4.

4.4 Image pair comparisons

In the first of the four comparisons, more than 90% of users selected "Fuzzy Red" as better than "Fuzzy Green" to convey an intuitive idea of risk. This seems to confirm that the colour red is indeed more associated with the visual perception of a threat.

In the second comparison, a majority (57.4%) of respondents agreed that "Fuzzy Red" was more appropriate than "Red with Texture" to visualize risk. This result suggests that a borderization using a gradual colour transition rather than a hard border, be it intrinsic or extrinsically superimposed, may communicate information about spatial risks more effectively. The third comparison seems to further confirm these findings: according to 83.6% of users, "Limited Fuzzy Border" is more appropriate than "Single Hard Border" for risk visualization. As the "fuzzy" borderization was deemed more appropriate in both case studies, it can be argued that this type of visualization can be effective in mapping various kinds of spatial hazards.

Somewhat surprisingly, 68.9% of respondents in the last comparison selected "Layered Border" as more useful than "Total Fuzzy Border" to understand the air

pollution distribution across the mapped area. This result, however, does not necessarily disprove the aforementioned findings. In fact, it may simply suggest that users still need some sort of "border" in the map to be able to gauge risk levels effectively and, while a fuzzy border with a "limit" seems to work better than a single hard border, a gradient extending all throughout the map surface may only increase confusion. This result reflects the effects on confidence shown in Subchapter 4.2. (Fig. 4.10)

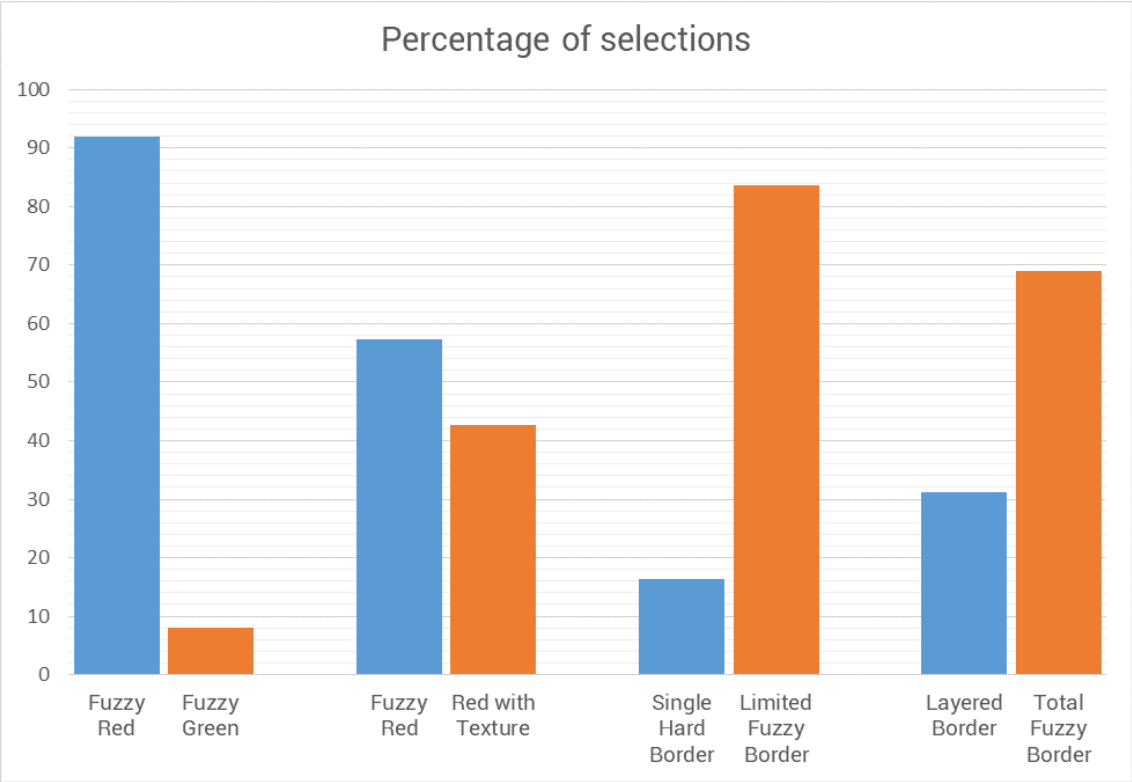


Figure 4.10: Results of image pair comparisons.

4.5 Ranking of colour shades

In a colour scale from red to yellow to green, an overwhelming majority of users indicated red as the shade associated with the highest level of risk. There was less agreement on the other end of the scale, even though most users still associated one of the two shades of green with the lowest risk. At least 50 users ranked colour shades from red (high risk) to yellow (medium risk) to green (low risk).

In the colour scale from red to orange to light yellow, red was again associated with the highest level of risk and 53 respondents provided the same colour scale from red (highest risk) to orange (medium risk) to light yellow (low risk). Answers to the third colour scale showed a similar pattern, with most users ranking shades from red (highest risk) to violet (medium risk) to light pink (low risk).

These findings provide evidence that users tend to intuitively associate certain colour shades with the visual perception of a threat. Namely, respondents overwhelmingly associate bright red with risk and lighter hues, or opposite values like green, with lower risk. (Table 4.1)

	5 (high risk)				1 (low risk)
	55	/	/	/	6
	/	50	3	8	/
	/	3	54	1	3
	1	5	2	40	13
	5	3	2	12	39
	55	/	/	/	6
	/	55	1	6	/
	/	/	60	1	/
	/	6	/	53	2
	6	/	/	1	53
	53	2	/	/	6
	2	52	1	6	/
	1	1	57	/	2
	/	6	/	54	1

	5	/	3	1	52

Table 4.2: Associations between colour shades and risk levels. Each value refers to the number of users who assigned a risk category (columns) to a colour shade (rows).

However, some users displayed atypical ranking patterns. Five to six users in each scale ranked colours in the exact opposite order compared to the average. This may genuinely indicate different perceptions and associations between risk and colour shades, but it could also suggest that these users simply misunderstood the question and mistook “1” for “highest risk” and “5” for “lowest risk”.

Additionally, a minimal number of other users ranked colour shades very differently from the rest, with seemingly inexplicable patterns. These users might suffer from colour-blindness and their altered colour perception could explain such uncharacteristic ranking choices. Indeed, these answers were more prevalent in the first colour scale (red to green) than in the other two; this seems to corroborate the colour-blindness hypothesis, as the red-to-green scale is typically troublesome for colour-blind individuals. Nevertheless, their answers did not seem to affect ratings in the red and green avalanche maps significantly; in fact, such ratings remained mostly unchanged even after filtering out the answers from these users, as they were only a tiny minority. This suggests that, while users do associate different colours with different risk levels, a wise use of the legend may help offset potential related biases. Additionally, the order of the questions within the survey, with avalanche maps coming after the PM10 maps, may have made users more aware of the study subject and less intuitive in their answers. This, in turn, suggests that randomizing the maps within a study case and between study cases may yield different results.

4.6 Open-ended questions

PM10 maps

Answers to the open-ended questions seemed to confirm further a significant use of containment- and distance-related heuristics by the users in order to assign point ratings. Bearing in mind the pitfalls and challenges associated with this kind of analysis as described in Subchapter 3.1, it was possible to identify several common keywords, locutions or expressions that served to uncover the usage of relevant heuristics.

However, this usage was not homogeneous across all borderizations. Unsurprisingly, relevant phrases and keywords were most common in "Single Hard Border": twenty-one (out of thirty-one) and thirteen users, in the maps without and with uncertainty, respectively, reported having used some kind of distance- and/or containment-related heuristic to assign their ratings. Conversely, only three (out of twenty-nine) and four users made such claims in the two "Total Fuzzy Border" maps without and with uncertainty respectively. These findings reflect similar results mentioned in previous subchapters. In fact, point ratings did suggest a frequent use of biases related to containment and distance in "Single Hard Border"; the introduction of extrinsic uncertainty and the subsequent decrease in ratings and confidence also mirrors the decrease in keyword frequency in the open-ended questions for the corresponding map. Similarly, open-ended questions seemed to confirm that respondents did not make heavy use of containment and/or distance biases in "Total Fuzzy Border" and relied more on colours instead. (Fig. 4.11)

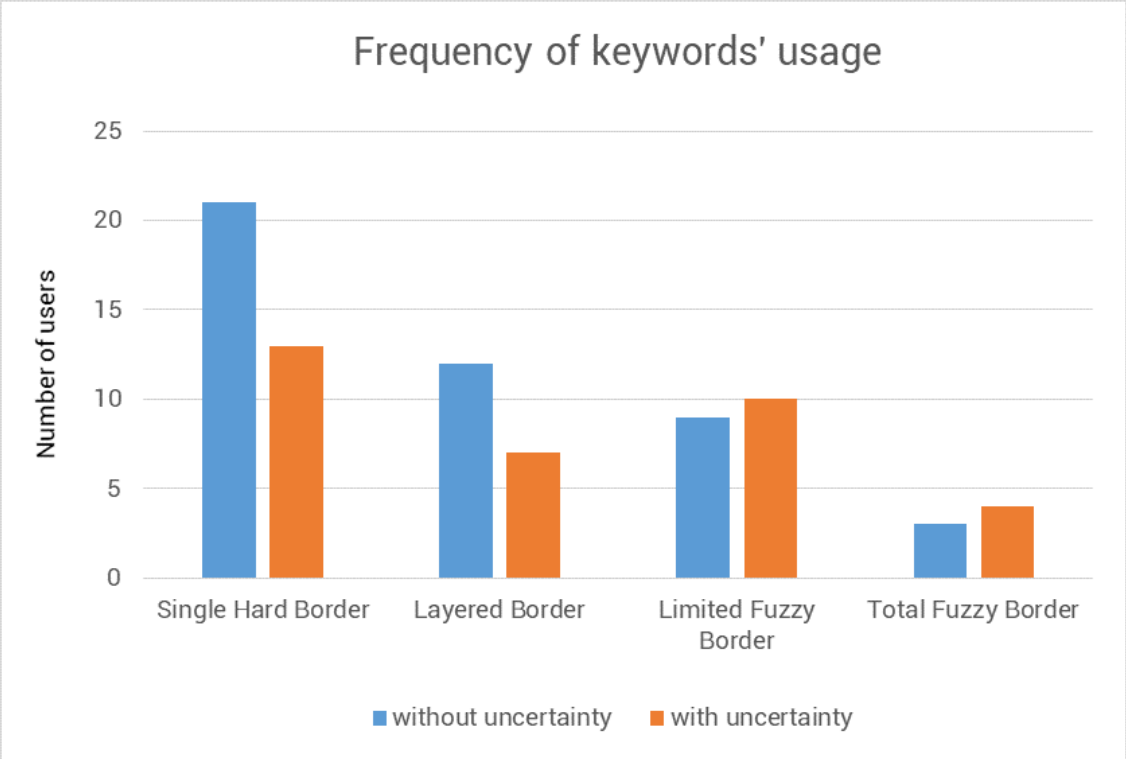


Figure 4.11: Frequency of usage of keywords and/or other relevant expressions in the open-ended questions.

Interestingly, the addition of extrinsic uncertainty caused a decrease in keyword usage in the "Hard Border" maps (single or layered), but it actually caused a small increase in the "Fuzzy Border" maps. This further suggests that the introduction of extrinsic uncertainty can cause users to shift their focus away from the original boundaries in the "Hard Border" maps. In contrast, it may provide them with a clear and straightforward border reference in the "Fuzzy Border" maps. However, even with extrinsic uncertainty, keyword usage in the "Fuzzy Border" maps and especially in "Total Fuzzy Border" remained low overall; this reflects aforementioned findings, which suggested that respondents mostly estimated risk levels in "Total Fuzzy Border" using changes in the background colour. A more in-depth analysis of the answers further supports this conclusion; several users reported that "Limited Fuzzy Border" helped them to assess risk levels effectively, whilst "Total Fuzzy Border" seemed only to increase confusion. (Tables 4.2 & 4.3)

Single Hard Border without uncertainty	"I've rate [sic] higher the points further away from high concentration of pollution." "I expect also that places outside the red zone will be less polluted going further away from the border." "B appears to be close to the enge [sic] but outside." "The closer the location gets to the high concentration areas, the less attractive it is" "Maybe they [C and D] are in the same condition, but I imagine the D one will be surrounded by more pollution" "Location A is very well within the low/no concentration zone" "Location A is the farthest from the red zone"
Single Hard Border with uncertainty	"C is the only location that falls into the non-risk zone and is not affected by uncertain data" "A and D are perceived similarly since they are both within this [uncertain data] area." "Location A – it is in the closer proximity to polluted area […] plus it is also under the uncertain area."
Layered Hard Border without uncertainty	"If it is closer to a border, I weighted [sic] it more towards whatever is across the boarder [sic]" "Location A is very close to the edge between low and moderate." "A is closest to the area of low PM10 concentration but is still within a moderate concentration area" "D slightly better than B because it lies closer to the limit of the high pollution zone" "The far [sic] from high pollution area, the cleaner the air"
Layered Hard Border with uncertainty	"C is too close to the uncertain area to be desirable" "The locations inside this uncertain data polygon are regarded as desireable [sic] at the same level as location B" "The location B is not desiderable [sic] at all because it is surely in a [sic] area with maximum pollution risk"

Table 4.3: Excerpts of answers to open-ended questions from the "Hard Border maps". These statements show the usage of containment- and distance-related biases.

Limited Fuzzy Border without uncertainty	"B and C are both in the diffuse region, but C seems rather outside, while B seems rather inside of it." "I find the location [A] too close to the polluted area"

	"The places get rates according to their distance from the <<hottest>> red area" "Best visualization for this topic, it shows the uncertainty" "A more graduated scale give [sic] more accurate informations [sic] and ratings naturally aligns [sic] with it" "Here I have more continuous data and I can be more confident in my choise [sic]"
Limited Fuzzy Border with uncertainty	"The areas in the box are quite Dangerous" "I feel more confident than with the previous map thanks to the black square" "B is very close to the uncertainty area, meaning that I am questioning how certain the data actually is for that area" "The locations within this [uncertain data] area are rated less desirable due to these uncertainties." "The places get ratings in accordance to their distance from the reddest area"
Total Fuzzy Border without uncertainty	"I tried to use the legend to gauge how polluted the area is" "I felt like all the locations were in bad areas due to the subtle gradient" "The further away from the dark red area, the better" "The whole visualization indicates an uncomfortable impression of all locations" "I see risks in all the choices" "Hard to distinct [sic] them quickly but still possible to see the gradient"
Total Fuzzy Border without uncertainty	"C is still too close to the uncertain area" "Again, the subtle gradient makes it difficult to work out the differences" "The uncertainty of the data makes the ranking of these four places more complex" "The uncertain data polygon does not affect the overall impression of not detecting differences in PM concentrations."

Table 4.4: Excerpts of answers to open-ended questions from the "Fuzzy Border" maps. The answers show the usage of relevant heuristics as well as the impact of different borderizations on users' experience.

Furthermore, some users reported that they found a gradient border to be more intuitive simply due to the nature of the hazard represented. One user answering

to the open-ended question under “Single Hard Border without uncertainty" wrote, “*I do not know a lot about air pollution, but I suspect that there is no such a clear delineation between areas of high air pollution and areas of low air pollution*". Another user within the same question wrote, “*Location B is located close to the high concentration zone, which might make it a bit less desirable, since a sharp boundary between high and low concentration seems unlikely*". Other users commented that points outside the high-risk area might also be affected by pollution as winds could blow from that direction. In fact, as seen in Table 4.3, users found a limited gradient not only to be more informative, but also more representative than a single hard border. As previously mentioned, the same was not true for “Total Fuzzy Border", where some users also seemed to have trouble understanding what the uncertain area truly meant as they had no clear reference to place the locations inside any risk category. Additionally, they sometimes seemed to understand the gradient in the map as a representation of uncertainty itself; therefore, the addition of extrinsic uncertainty caused additional confusion for them. Finally, several users reported that the “Total Fuzzy Border" map somewhat increased across the whole map space the visual perception of a threat and subsequent feelings of risk avoidance. This finding may suggest the presence of the affect heuristic, as map readers might rely on immediate feelings to formulate their judgements.

Avalanche maps

Answers to the open-ended questions showed that survey respondents did use containment and distance to drive their rating choices in the avalanche maps as well. However, usage frequency did not seem to display significant variations across different borderizations; this result reflects the distribution of point ratings as seen in Subchapter 4.1.3, with few clearly detectable patterns. Simultaneously, distance from the border appeared to be an extremely relevant factor in assigning ratings. As one user wrote under “Fuzzy Red" referring to a location outside the risk area but close to its border, “since there may come an avalanche that is bigger than everything before, I would probably not buy either". Interestingly, one user under “Fuzzy Green" reported the exact opposite feeling writing,

“as avalanches are limited by typology or terrain it is unlikely that the safety [*of a location*] will decrease, or that being slightly closer is of a higher danger if it has never been affected before".

As in the PM10 maps, answers to the open-ended questions under the avalanche maps also provided insights about the representativeness of the borderizations. Firstly, the use of green to represent danger appeared somewhat questionable. Many users reported that it felt confusing and inappropriate, although they also seemed able to make use of the legend to offset such perceptions and come up with map judgements more confidently. As one user wrote under “Fuzzy Green”, “the colour scheme associates with a positive event not a disaster thus it creates biases in my mind. But I was attentive to the legend."

Extrinsic visualization of uncertainty also seemed to produce conflicting results. Some viewers reported that, as in the PM10 maps, a hard border was unrepresentative of an avalanche; as one user wrote under “Green with Fog”, “the border is clearer. But not realistic because I'm reality [sic] you wouldn't have this sharp defined border". However, the same user reported that it “helps in terms of decision making". Another user under “Red with Fog" wrote that “all choices were made sorted hierarchically based on a distance from clear boundaries of a high risk zone". Two users under the following map, “Red with Layer", also wrote that “this time the uncertain area is more understandable" and “this visualization of uncertainty is much clearer and feels easier to access", while claiming that their overall logic behind the ratings was the same as in “Red with Fog". These findings might suggest that, while a hard border was deemed as less accurate than a gradient, it also helped decision making by triggering the containment and distance heuristics; among the two hard borders used, a texture layer seemed more readable than a foggy layer.

In fact, several users also reported that their perception of the points inside and outside the high-risk area changed after the introduction of extrinsic uncertainty. Interestingly, one user under “Red with Texture" wrote “A looks almost less safe than D as it has both red and pattern markings". Location A as portrayed in the original map lay on the uncertain area within the high-risk area,

while D lay completely inside the high risk area. In other words, the use of a texture layer seemed to offset the distance heuristic and increase the feeling of threat associated with A. Another user under “Red with Fog” wrote, “the uncertainty doesn't improve the eevaluation [sic] of safety [for point A] as there is still a high likelihood of avalanches” and then added “the uncertainty drastically decreases the assessment of safety [for point B] as there is an increased risk associated with it”. A user under “Green with Fog” wrote, “only E seems to be a safe place”, while another user reported that “I feel safer in E now than before”; point E in this map was the furthest one from the high-risk area.

In contrast to the PM10 maps, these observations suggested a pattern of risk aversion. In fact, the addition of extrinsic visualization to the map seemed not only to “dichotomize” the space between safe and unsafe, but also to increase risk perception both in the high-risk areas and in the uncertain ones lying immediately outside. Therefore, users felt compelled to avoid points they had previously viewed more positively and they only felt fully safe in the safest point. However, point ratings did not seem to support such hypotheses; as previously seen, these ratings showed few significant variations across different borderizations.

The aforementioned answers, along with a few others from all borderizations, are listed in Table 4.4 below.

Fuzzy Red	“A has still some risk, while D is just outside the risk area. Since there may come an avalanche that is bigger than everything before, I would probably not buy either.” “E is slightly deeper into the red area, which is why it received the lowest rating” “I would like to know the distance of point B to the high risk area before feel [sic] safe”
Fuzzy Green	“The further away you are from the greenzone the safe [sic] it is” “B and D are not affected at all. I feel more unsure about D however because it is closer to the risky area” “B and D) are unaffected by avalanches, as avalanches are limited by topology and terrain it is unlikely that their safety will decrease,

	or that being slightly closer is of a higher danger if it has never been affected before." "The farer [sic] I am from the risk zone, the safer I feel" "The colour scheme associates with a positive event not a disaster thus it creates biases in my mind. But I was attentive to the legend. I am afraid that another unexperienced map user might be confused. Not having distinct boundaries just blurred circles also keeps me from making an unambiguous choice"
Red with Fog	"[for point C, outside the high risk area] the uncertainty is greater than first predicted. You could also be in the red" "A) The uncertainty doesn't improve the eevaluation [sic] of safety as there is a high still a high likelihood of avalanches. B) The uncertainty drastically decreases the assessment of safety as there is increased risk associated with it" "Distance from the risk zone remains relevant" "All choices were made sorted hierarchically based on a distance from clear boundaries of a high-risk zone"
Green with Fog	"I tend to lower rating in the uncertain area" "B seems to lie outside both risk and uncertain area, but not very far. Thus A is the only safe bet" "Green is a weird color to express danger" "[referring to point C] the uncertainty makes me perceive it as riskier" "Risk area (green) and uncertain data have the same choice for me" "The border is clearer. But not realistic because I'm [sic] reality you wouldn't have these sharp defined borders. But it helps in terms of decision making" "Note – the green color may confuse a non-expert into thinking that the area might be free from danger as the green usually is associated with such idea" "Locations A-C are supposedly safe, but the further away from the risk area, the better"
Red with Texture	"Same reason as the previous map, only this time the uncertain area is more understandable" "A almost looks less safe than D as it has both red and pattern markings" "I consider the uncertain area as if it was risk area"

	"A is in risk area and uncertain area at the same time, which makes it not safe at all" "Striped polygon makes the area appear unsafe"
Green with Texture	"I feel safer in e now than before" [note: point E lay in the safe area without uncertainty] "I don't believe in the uncertain area, it might be unsafe, so I will avoid it" "My opinion about C change [sic] with this visualization…Now I can feel that it is quite close to the uncertain area so I feel a little bit less safe" "Only E seems to be a safe place" "The key makes risk very clear so easy to answer"

Table 4.5: Excerpts of answers to open-ended questions from the avalanche maps.

4.7 Summary of results

Use of heuristics and reasoning on uncertain geospatial data

- Containment: the present work *confirms* previous literature findings, as results show evidence of some use of the containment heuristic. With hard borders, respondents tended to treat map spaces dichotomically and use these boundaries as semantic divides to gauge risk levels and assign point ratings. Conversely, in the absence of such borders users mostly relied on colour shades to assess risk levels and assign ratings.

- Distance: the present work *confirms* previous literature findings, as results show evidence of some use of the distance heuristic. Respondents used distance from a border as the main metric to assign point ratings in the absence of any other relevant information about risk levels. Distance was often even more of a decision-making factor than colour. However, the exact border used as a mental reference to calculate the distance could vary depending on the visualization used.

- Representativeness: the present work *somewhat confirms* previous literature findings, as results show evidence of some use of the representativeness heuristic. Respondents claimed to assign ratings differently when map stimuli were visualized in ways that felt representative of the phenomenon – e.g., a fuzzy gradient for a PM10 map or a red area to show elevated risk levels. The aforementioned containment bias can also be understood as an extension of the representativeness heuristic. However, not all representative maps elicited the same responses, and some non-representative maps were actually preferred to some representative ones which were judged less appropriate (see Subchapter 4.6.2).

- Availability: the present work *neither confirms nor disconfirms* previous literature findings. The results did not show evidence of use of the availability heuristic; however, it is unclear whether the result is only due to this heuristic not being relevant for this particular subject.

- Adjustment to an anchor: the present work *somewhat confirms* previous literature findings, as results showed evidence of some use of the adjustment-to-an-anchor heuristic. A few users seemed to assign point ratings basing on the first map they had viewed. This pattern was much more widespread in the avalanche maps, where many users had likely become aware of the study subject, than in the PM10 maps where only a minority of respondents seemed to make use of the heuristic. Results also showed that the effects of this heuristic can be controlled through appropriate techniques for map legends and user testing.

- Other heuristics: the present work *somewhat confirms* previous literature findings, as point ratings in PM10 maps showed some evidence of loss aversion patterns. However, results also suggest that these patterns might only be triggered by certain visualizations and topics. Indeed, open statements in avalanche maps seem to provide support for the presence of risk aversion rather than loss aversion patterns. At the same time, point

ratings in the same maps do not reflect this hypothesis. Additionally, results might suggest the presence of the affect heuristic.

Relation between visual variables, uncertainty visualization techniques and heuristics

- "Hard Border" vs. "Fuzzy Border": the present work *does suggest* that altering these borderizations can help control and reduce heuristic use. In the PM10, visualizing pollution levels through a "fuzzy" border helped introduce more nuanced judgements and complex levels of risk perception across the map space. However, results from the avalanche maps were more ambiguous. In fact, while open-ended statements did suggest that a "fuzzy" border helped reduce heuristic use, the actual point ratings did not support this hypothesis as they showed little significant change between different borderizations.
- "Single Hard Border" vs. "Layered Hard Border": the present work *does suggest* that altering these borderizations can help control and reduce heuristic use. In fact, users appeared to be less biased by a containment heuristic when assigning ratings in "Layered Hard Border"
- "Limited Fuzzy Border" vs. "Total Fuzzy Border": the present work *does suggest* that altering these borderizations can help control and reduce heuristic use. However, it is notable that a reduction in heuristic use does not necessarily coincide with an improved map experience. In fact, results showed that, while "Total Fuzzy Border" reduced heuristic use by removing any border from the map, it also elicited somewhat negative responses by users. In the open-ended statements, respondents reported that "Total Fuzzy Border" did not help them in decision-making and the subtle gradient felt confusing and inappropriate to represent the phenomenon.
- Intrinsic vs. extrinsic uncertainty: this present work *does suggest* that altering the visualization of uncertain data within the map can help control and reduce heuristic use. However, the specific effects of uncertainty visualization are ambiguous and somewhat inconsistent. In PM10 maps, the

introduction of extrinsic uncertainty reduced the containment and distance biases when pollution boundaries were visualized through a hard border, as the uncertainty added information for more complex reasoning. Conversely, in “fuzzy” maps and especially in “Total Fuzzy Border” extrinsic uncertainty mostly seemed to add confusion. In the avalanche maps, users reported that extrinsic uncertainty made the map more readable and aided decision-making, but it also seemed to trigger distance and containment heuristics. However, point ratings remained mostly stable as previously showed. In any case, extrinsic uncertainty consistently increased perceptions of risk and potential damage across all visualizations. This can be framed as a manifestation of the affect heuristic as outlined by Raue & Scholl (2018): users might have associated uncertainty with feelings of avoidance and threat regardless of the actual information that the visualization carried. See Table 4.6 and Table 4.7 for an overview of statistical testing results for each pair of maps and map sets.

	A	B	C	D	Conf.
With vs. without uncertainty					
Hard vs. Fuzzy					
Hard with vs. without uncertainty					
Fuzzy with vs. without uncertainty					
Hard vs. Fuzzy (with uncertainty)					
Hard vs. Fuzzy (without uncertainty)					

Table 4.6: Overview of statistical testing results for average point ratings and confidence levels in PM10 maps. Light green indicates significance for p<0.05, dark green for p<0.01.

	A	B	C	D	E	Conf.
Fuzzy Red vs. Red with Texture						
Fuzzy Green vs. Red with Fog						
Fuzzy Green vs. Green with Texture						
Green with Texture vs. Green with Fog						

Table 4.7: Overview of statistical testing results for average point ratings and confidence levels in avalanche maps. Light green indicates significance for p<0.05, dark green for p<0.01. The table only includes pairs with at least one significant result.

- Colours: this present work *does somewhat suggest* that altering colour choices to visualize natural hazards and the associated uncertainty can help control and reduce heuristic use. Respondents agreed that they intuitively associated certain colours, namely red and orange, with intuitive ideas of 'risk' more than others, such as green. Therefore, visualizing this kind of data through uncharacteristic colours can uncover patterns of heuristic use (e.g., representativeness heuristic). However, point ratings in the avalanche maps did not seem to significantly support this hypothesis, as they remained mostly stable throughout all the visualizations regardless of the colour used.
- Additional observations: females and users with average or low levels of expertise with maps appeared to be more conservative and less confident in their ratings compared to the other users. However, the relatively low number of users within each single category, as well as their uneven distribution, made any solid statistical analysis difficult. Furthermore, while absolute ratings were different, their variations between maps did reflect the same patterns as those from other users. Future surveys could focus more on these personal characteristics in order to try and determine whether such differences have any factual basis.

User test for visualization evaluation

- Investigation method and platform: an online survey proved effective in gathering a sufficiently large and diverse set of participants. Anonymity not only eliminated any ethical concerns, but it also guaranteed that users did not feel pressured to answer in any specific way. The online tool SoSciSurvey offered a comprehensive and flexible environment to build such a survey and analyse its results.
- Randomization: randomizing the order of the points in the map was useful to control for possible effects of the adjustment-to-an-anchor heuristic, although some users did report that they automatically sorted locations in the same order across all visualizations. Dividing the participants

into two randomized groups also kept the survey from becoming excessively long, while still making it possible to include all the maps in the questionnaire. However, results from the avalanche maps seem to suggest that users had become aware of the study subject by that point and were less influenced by intuitive heuristics than before. Therefore, a future similar study could also benefit from a randomization of the order of the study cases.

- Question structure: keeping questions simple and repeating them throughout the whole survey effectively seemed to help users navigate through the questionnaire while maintaining results' comparability. Likert-type scales were also useful to turn answers into numerical data and subsequently perform statistical testing with relative ease. Open-ended statements and colour shades rankings provided additional insights on users' reasoning and logical processes; however, the answers that emerged from those sections were not necessarily consistent with other numerical results. A future similar survey might need to be built slightly differently in order to investigate the potential motives behind any discrepancies in different data categories.
- Limitations and further improvements: while this survey seemed to be overall effective for the study, it did suffer from some minor shortfalls that can be taken into consideration for a future survey. For instance, an eye-tracking technology could potentially provide further insights on users' attention and focus patterns, as well as additional information on timing and decisions under time pressure. Provided a large enough number of users is available, a one-to-one free interview or a focus group could also yield interesting results in a more qualitative study.

5. Conclusions and outlook

This work had the primary goal to investigate how map-readers, be they experienced or inexperienced, make use of heuristics to decode and reason upon cartographic information in uncertain contexts. With this aim, an extensive literature review served as a basis to identify study cases, design several maps from the concept of "borderization" and finally build a user test aimed at enabling such research. Subsequent findings were interpreted in light of the original research objectives, as well as the theoretical state-of-the-art concerning heuristics and cognitive biases in geospatial visualization.

Results from the test seemed to further validate previous literature findings by highlighting the use of several different known heuristics in many ways and contexts. Namely, users appeared to consistently adopt containment and distance heuristics to judge risk levels of different points in space. Results also proved that some visualization techniques, such as a gradient instead of a single hard border, can help reduce these heuristics and come up with different judgements, which however may or may not be accurate. Additionally, the test proved that uncertainty visualization also plays a significant role in user choices and perceptions, mainly by affecting their confidence and by changing heuristic use in numerous and sometimes contrasting ways. In other words, uncertainty visualization acted as an added layer of complexity that made choices less straightforward overall, thus also complicating the use of heuristics. Finally, results suggested that the choice of colours does affect users' perception of the maps, although an effective use of the legend can offset any such impact.

Findings also provided evidence for the use of some of the original heuristics cited by Tversky & Kahneman. Arguably, containment and distance biases can also be framed as evidence for these heuristics. Additionally, users seemingly adopted other heuristics beyond the original ones introduced in 1974, such as affective association and loss aversion.

An online questionnaire proved to be an effective tool to recruit many users in a short time and collect as truthful and genuine answers as possible. However, the questionnaire presented some limitations, such as the inability to track users' eye movements to gather insights about map focus, or the impossibility to interview respondents in real-time. Additionally, a similar survey might benefit from the randomization of study cases, since results from the test may suggest that users collected knowledge and awareness from the first study case and, in the second study case, did not employ heuristics that they might have adopted otherwise.

Notably, it was not possible to identify fully quantitative criteria to detect heuristic usage. Due to the very nature of the phenomenon, any numerical finding needed to always be supported by some degree of qualitative analysis in order to produce meaningful conclusions. Significantly, past literature also does not provide standardized quantitative results to use as pre-conditions to detect heuristic usage. Future cartographic research in the field of heuristics might need to work more closely with other cognitive science domains in order to identify unequivocal criteria and remove as much uncertainty and ambiguity as possible.

Further work in the field can benefit from the findings of this study in several ways:

- By choosing appropriate cartographic designs to draw boundaries, through techniques that are as consistent as possible with the subject, scope, and data type of the map,
- By building heuristics-aware natural hazard maps that not only communicate the risk effectively, but also feel helpful and easy to read for final users, and
- By increasing its collaboration with cognitive science domains to identify yet unknown links between heuristics, biases and common visualization methods.

Overall, the present work shed further light on cognitive biases and human reasoning in cartographic visualization and, more specifically, on how humans use

logical shortcuts to analyse geospatial information in uncertain contexts. These findings sit in an underexplored domain that, however, remains crucial for cartographers to come up with better visualizations and bridge the gap between data, maps and users. As maps are primarily a communication tool, it is imperative to take heuristics and cognitive biases into account to provide users with the best possible information and support decision making by avoiding unnecessary confusion.

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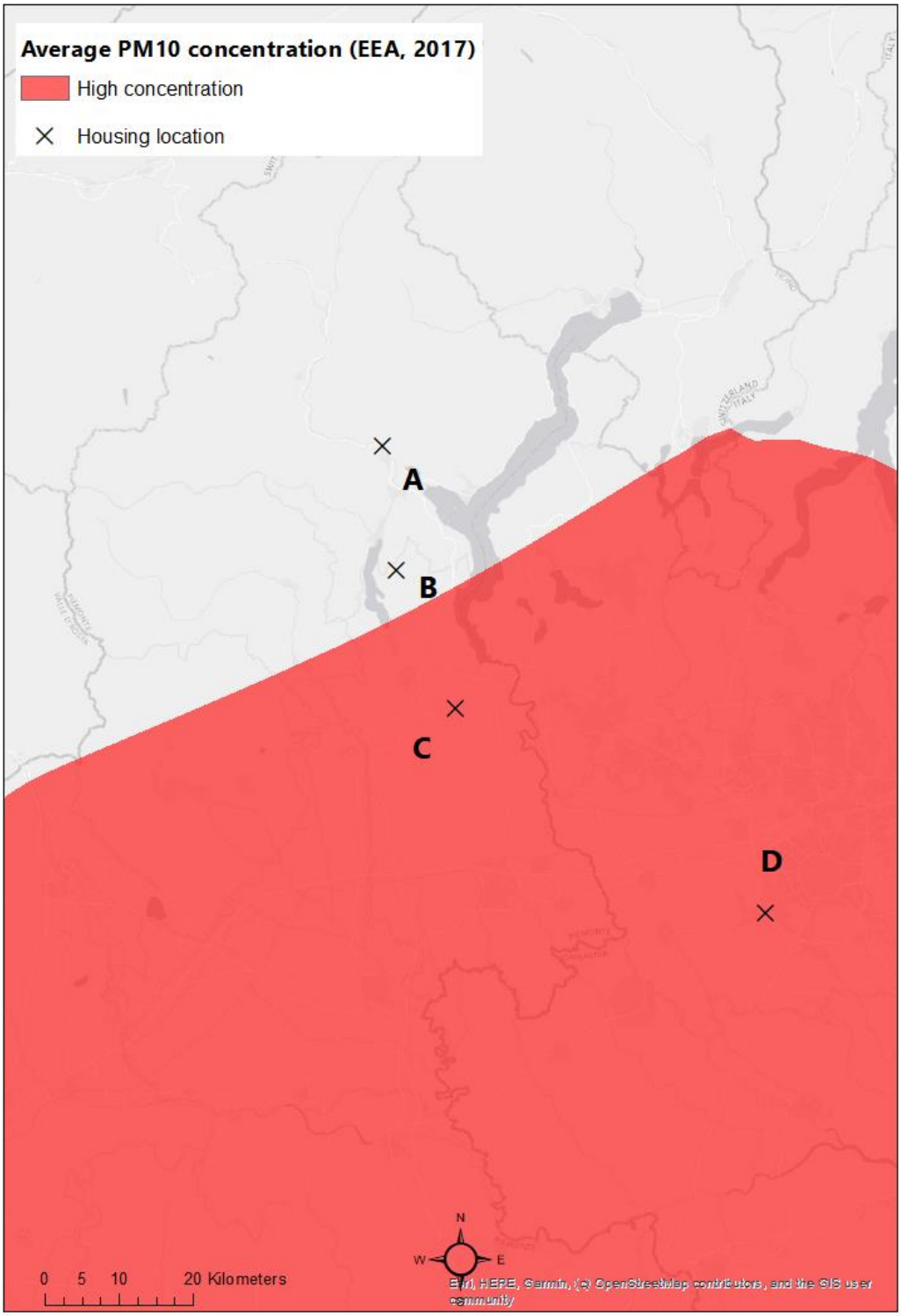
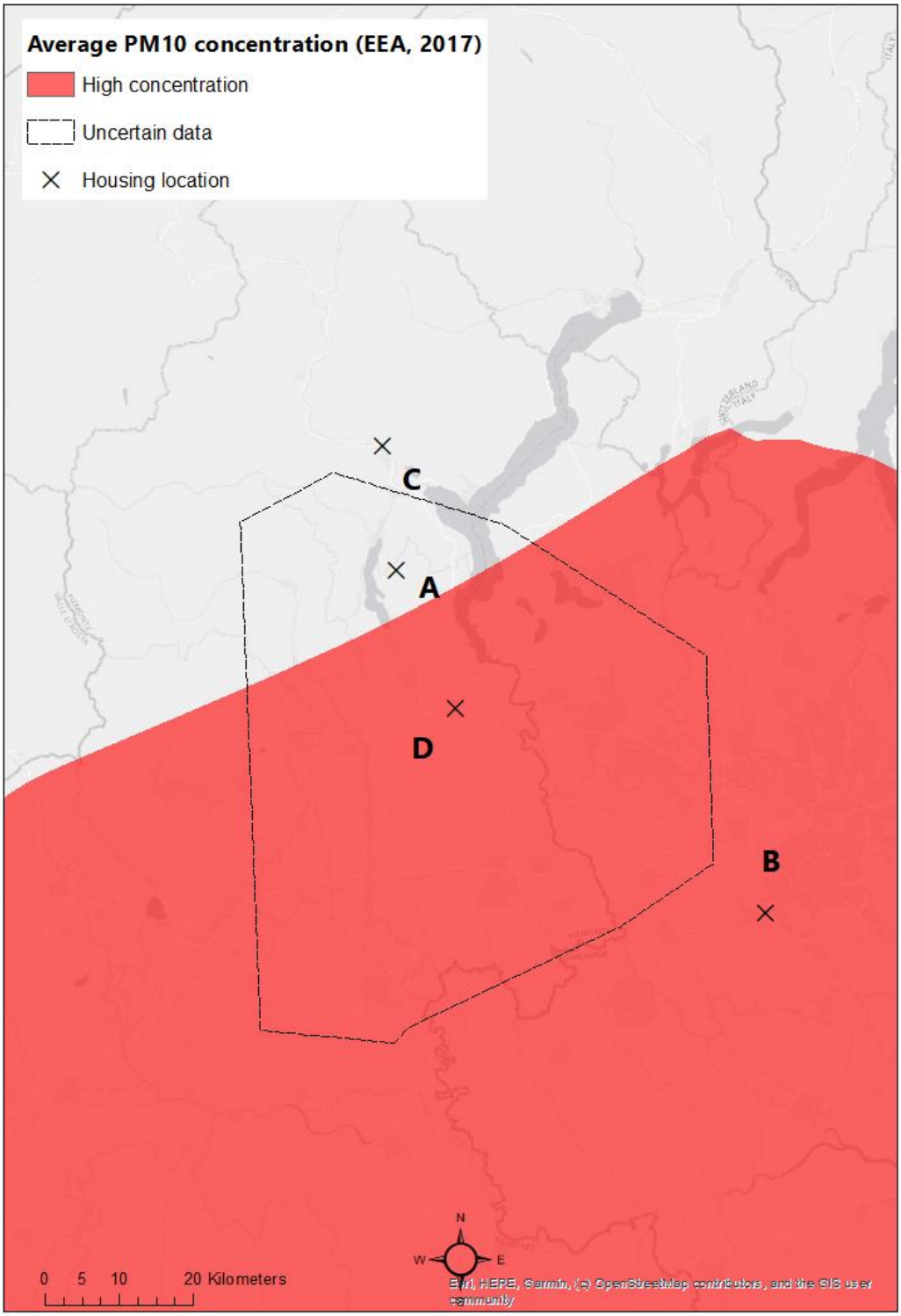
Appendix: Borderizations

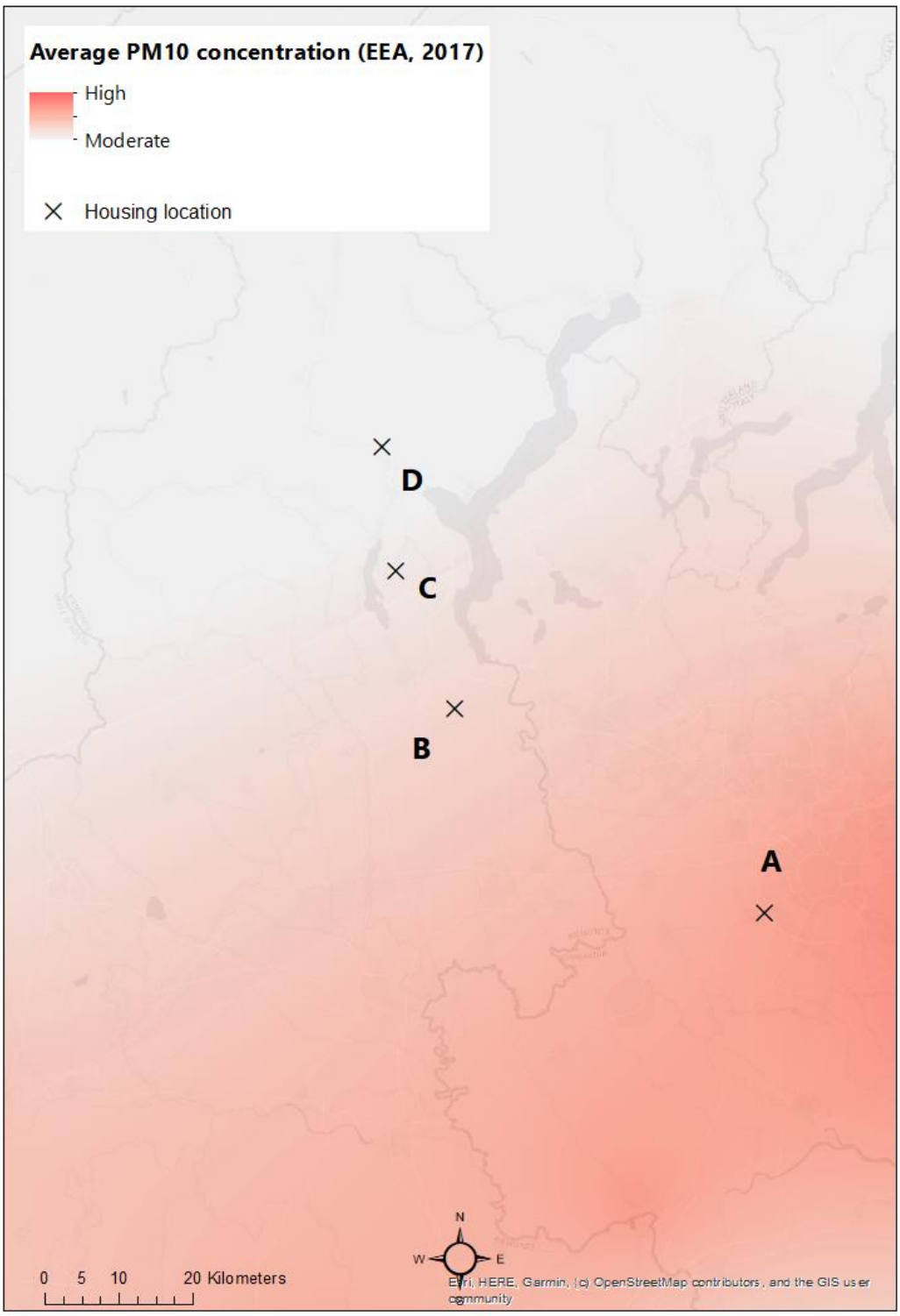
Single Hard Border



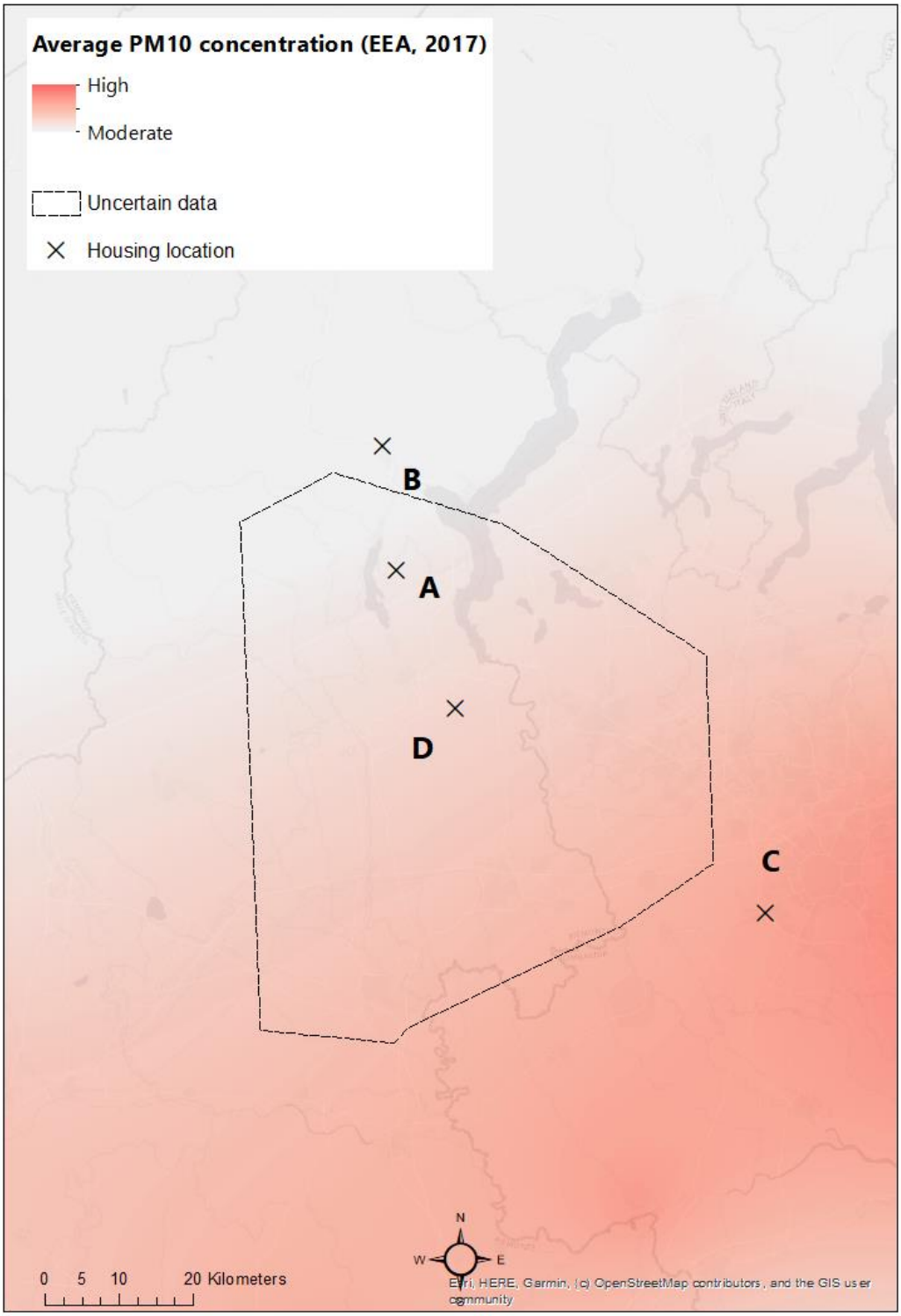
Single Hard Border with extrinsic uncertainty



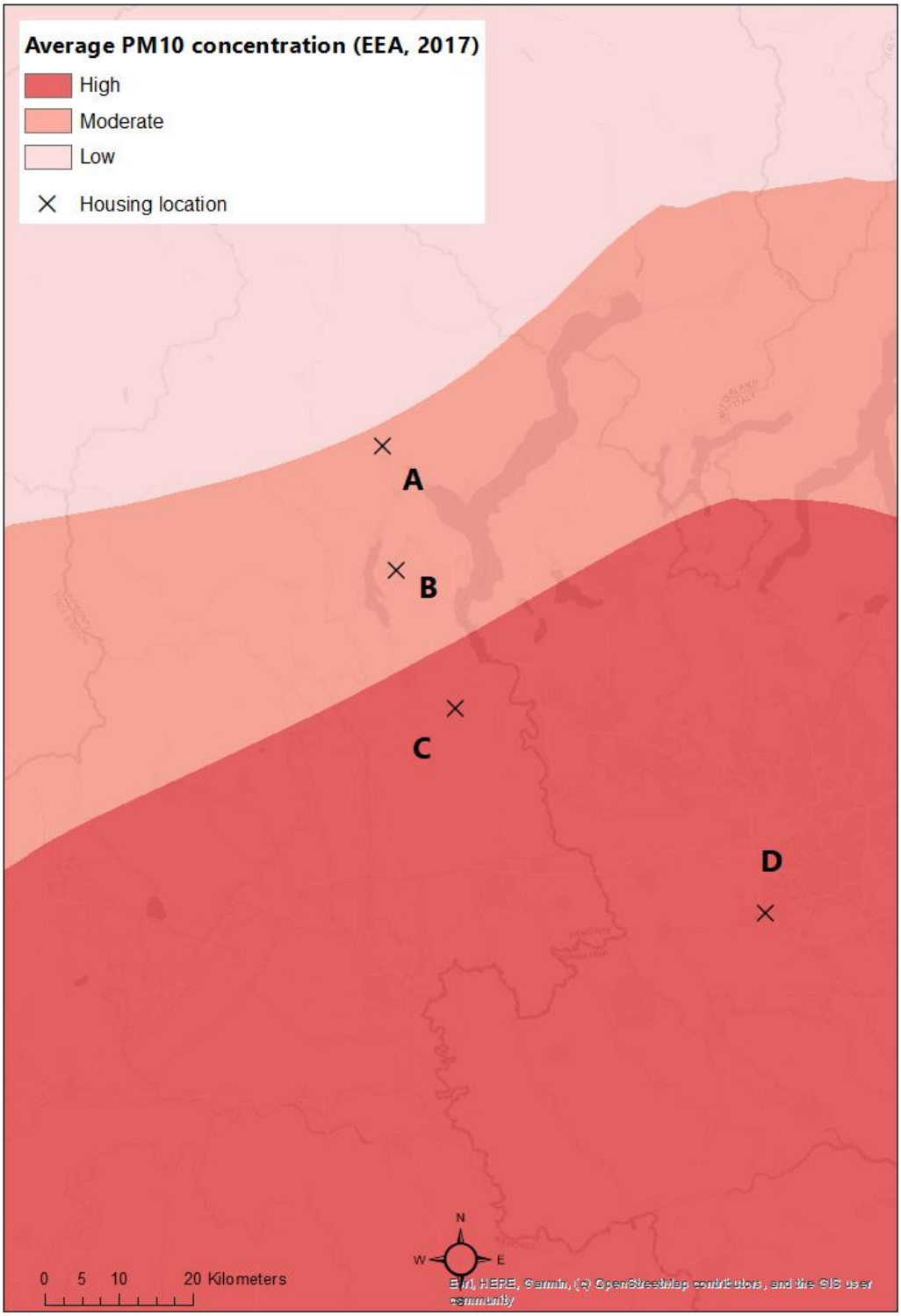
Limited Fuzzy Border



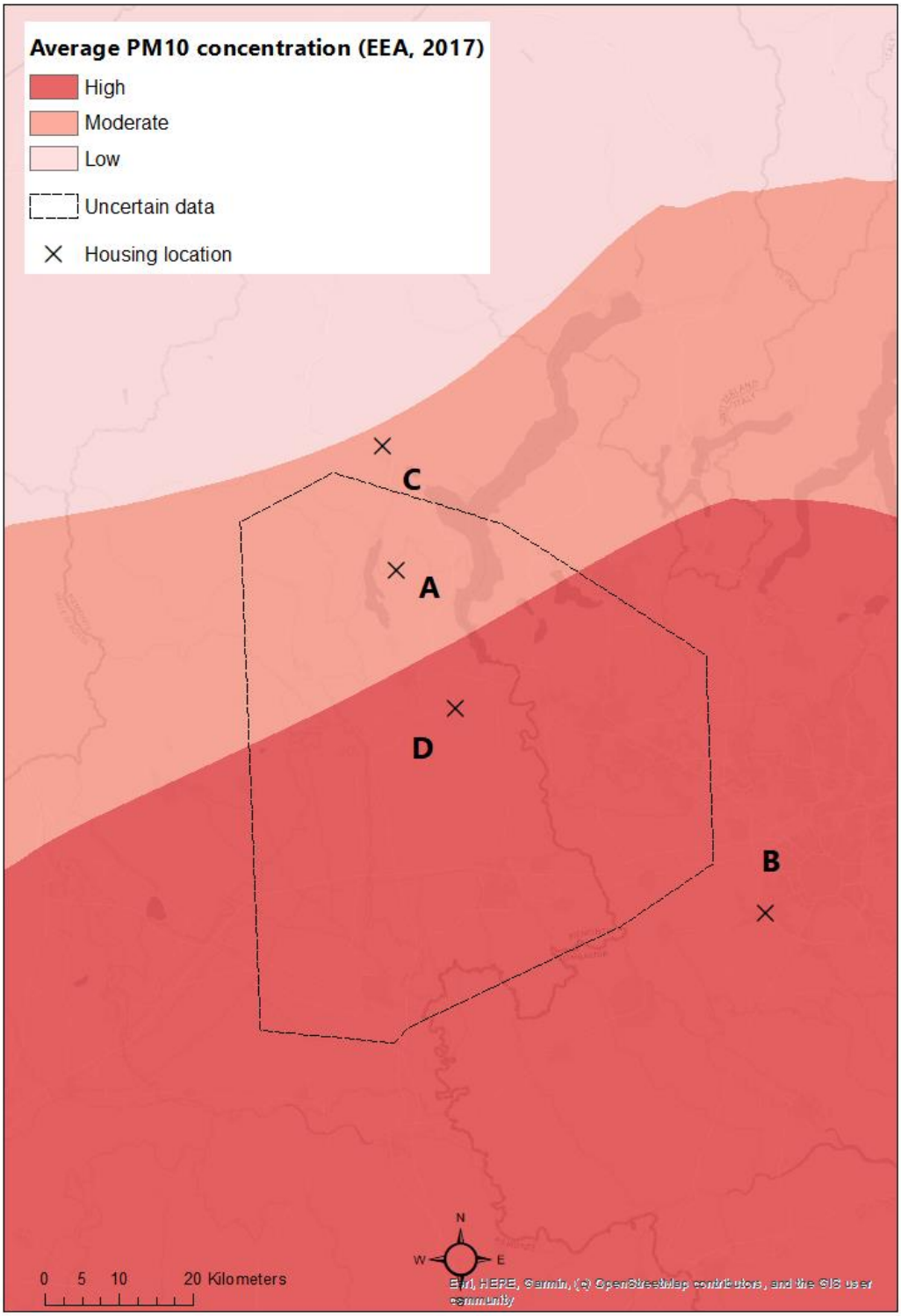
Limited Fuzzy Border with extrinsic uncertainty



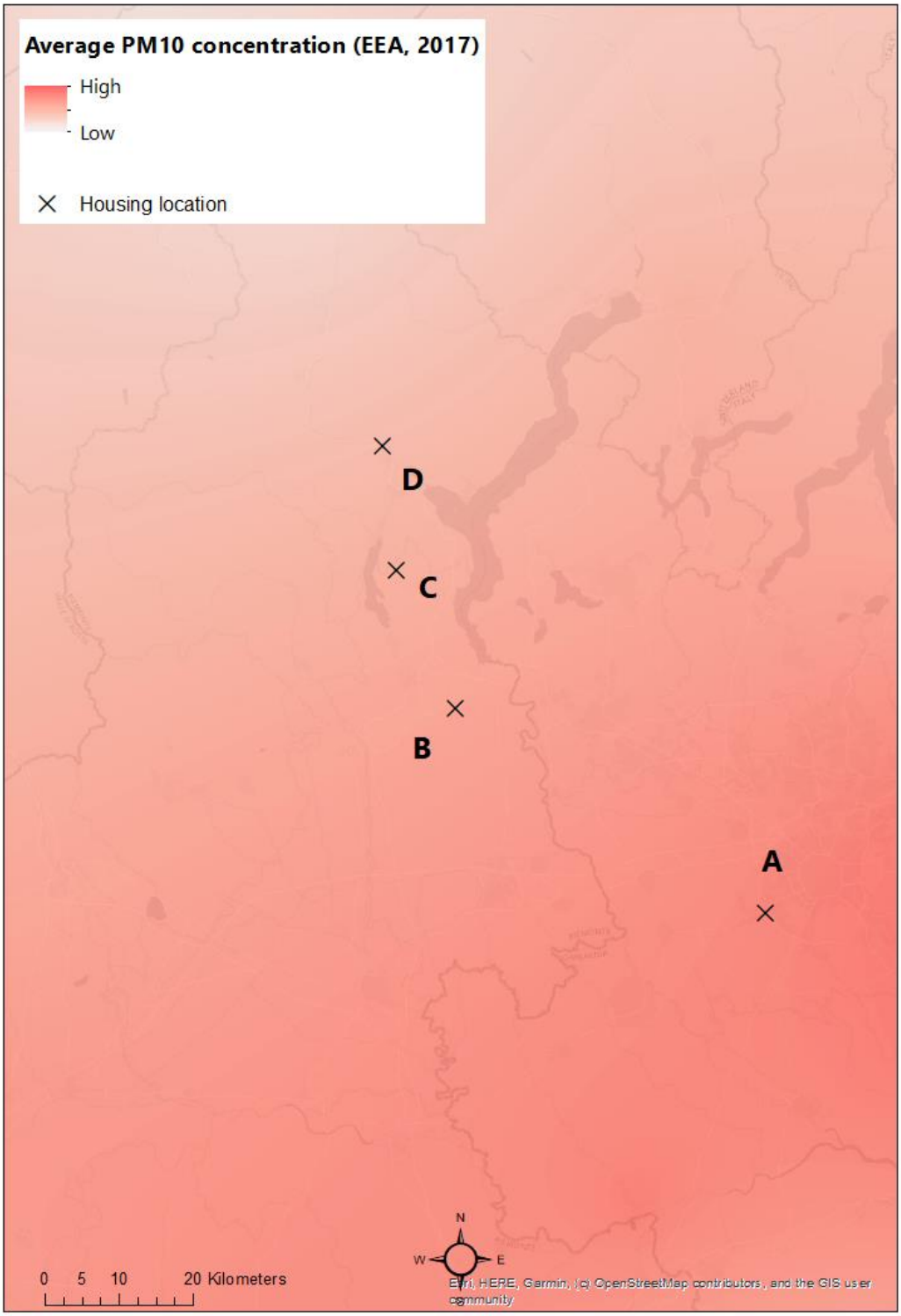
Layered Border



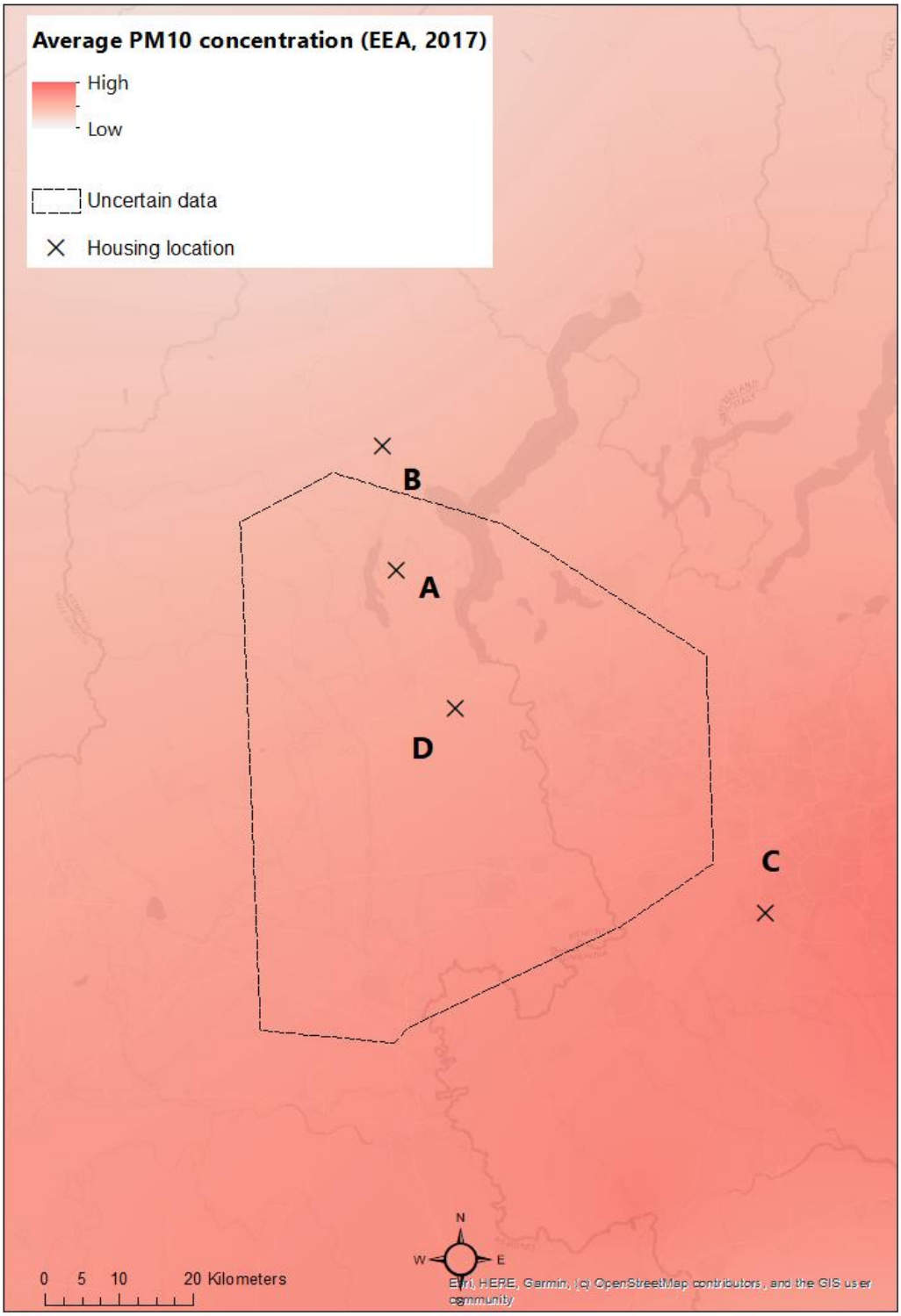
Layered Border with extrinsic uncertainty



Total Fuzzy Border



Total Fuzzy Border with extrinsic uncertainty



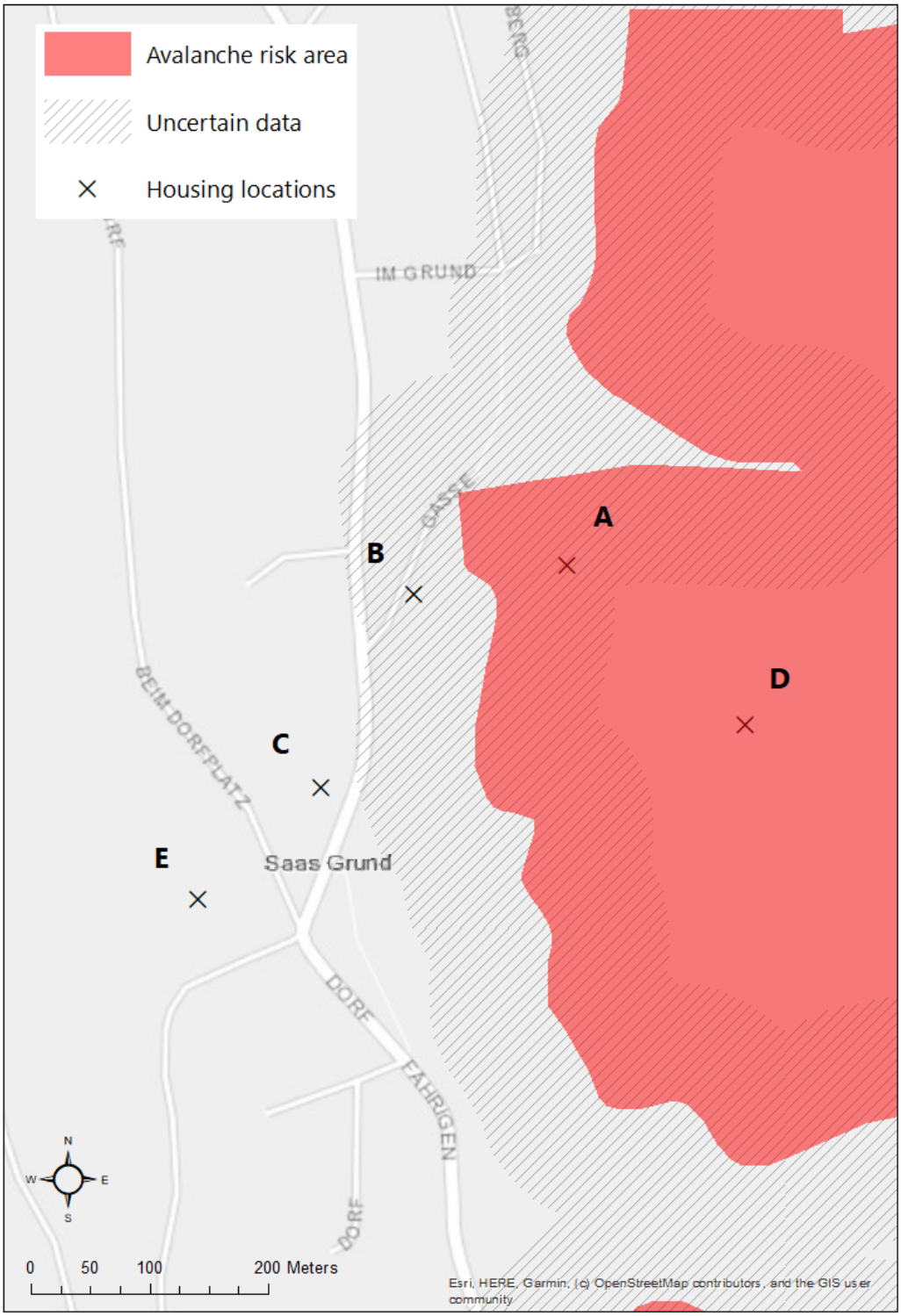
Fuzzy Red


Avalanche risk area
Housing locations
DORF
IM GRUND
BERG
GASSE
A
E
C
D
B
BEIM DORFPLATZ
Saas Grund
DORF
FAHRIGEN
DORF
N
W
E
S
0
50
100
200 Meters
Esri, HERE, Garmin, (c) OpenStreetMap contributors, and the GIS user community

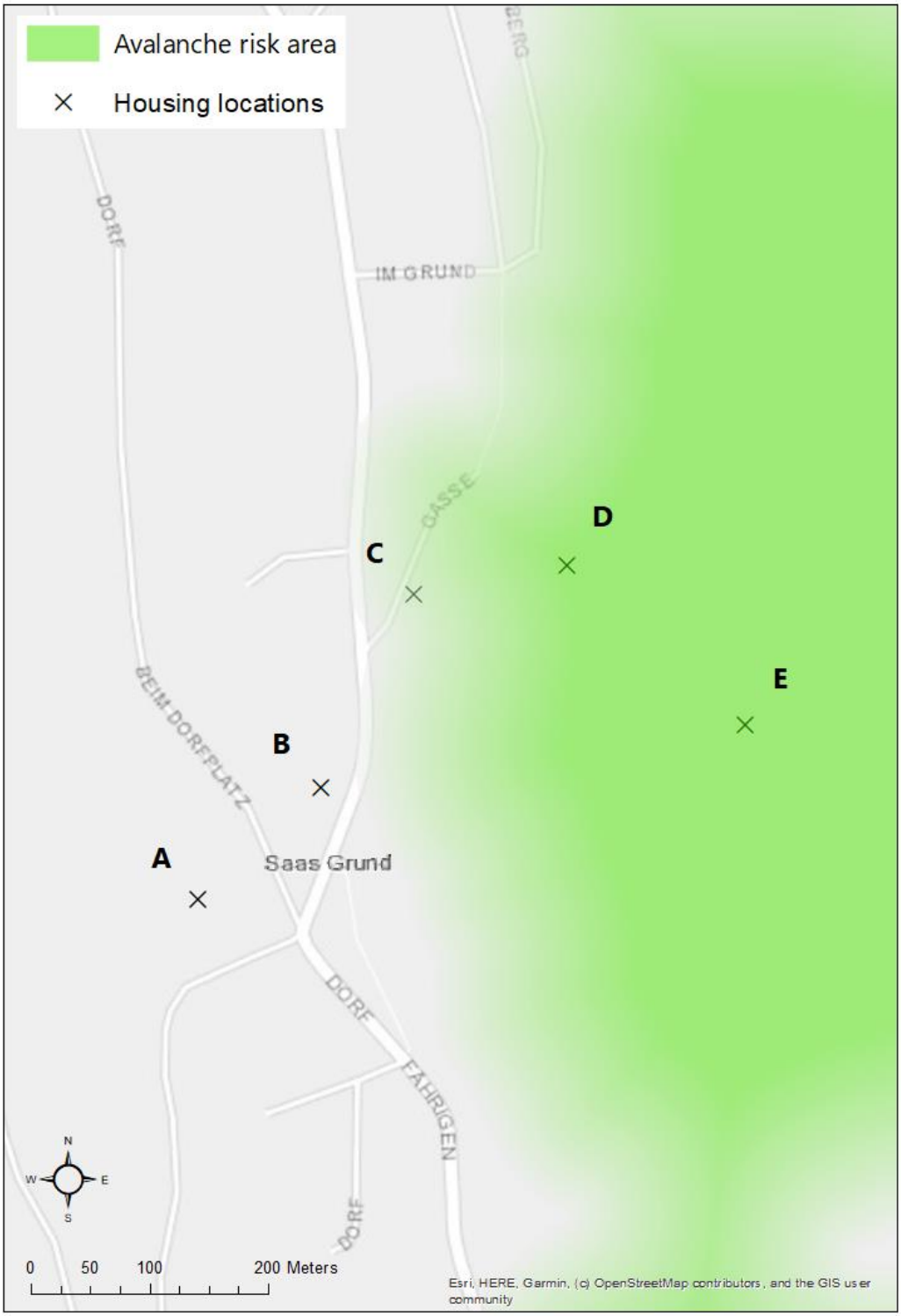
Red with Fog


Avalanche risk area
Uncertain data
Housing locations
IM GRUND
GASSE
BERG
BEIM DORFPLATZ
DORF
FAHRIGEN
Saas Grund
A
B
C
D
E
N
W
E
S
0
50
100
200 Meters
Esri, HERE, Garmin, (c) OpenStreetMap contributors, and the GIS user community

Red with Texture



Fuzzy Green


Avalanche risk area
Housing locations
DORF
IM GRUND
BERG
GASSE
BEIM DORFPLATZ
Saas Grund
DORF
FAHRIGEN
DORF
A
B
C
D
E
N
W
E
S
0
50
100
200 Meters
Esri, HERE, Garmin, (c) OpenStreetMap contributors, and the GIS user community

Green with Fog

Green with Texture